B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE, VIJAYAPUR-586 103

DEPARTMENT OF MATHEMATICS

First Internal Assessment

Sem: I **Sub:** Algebra-I and Calculus-I **(DSC1)** **Code:** 21BSC1C1MAT1L

Date: 02 – 12 – 2022 **Time:** 4:00 PM – 5:00 PM **Max. Marks: 30**

Q.No.1. **Answer Any Three Questions** 2×3=6

a) Find the rank of a matrix $A = \begin{bmatrix} 3 & -1 & 4 \\ 0 & 5 & 1 \\ 4 & -3 & -2 \end{bmatrix}$.

b) If φ for the curve $r = ae^{b\theta}$.

c) If $f(x) = \begin{cases} x^2+3 & \text{when } x \le 1 \\ x+1 & \text{when } x > 1 \end{cases}$, find $\lim_{x \to 1} f(x)$ if it exist.

d) Find the nth derivative of $\sin^3 x$.

Q.No.2. **Answer Any Three Questions** 4×3=12

a) Verify the Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$.

b) Write a matrix $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrix.

c) Derive the expression for angle between the radius vector and tangent at any point on the curve.

d) Find the slope of the tangent at specified point for the curve $r = a(1 - \cos\theta)$ at $\theta = \frac{\pi}{3}$.

Q.No.3. **Answer Any Three Questions** 4×3=12

a) If $\lim_{x\to a} f(x) = l$ and $\lim_{x\to a} g(x) = m$, then prove that $\lim_{x\to a}[f(x) + g(x)] = l + m$.

b) If a function $f(x)$ is continuous in $[a, b]$, then show that it is bounded in $[a, b]$.

c) State and prove Leibnitz's theorem.

d) Find the nth derivative of $y = \frac{1}{6x^2-5x+1}$.

B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE, VIJAYAPUR-586 103

DEPARTMENT OF MATHEMATICS

First Internal Assessment

Sem: I **Sub:** Mathematics-I **(OEC1)** **Code:** 21BSC101MAT1-A

Date: 02 – 12 – 2022 **Time:** 4:00 PM – 5:00 PM **Max. Marks: 30**

Q.No.1. **Answer Any Three Questions** 2×3=6

a) Define symmetric and skew symmetric matrix.

b) Find $\lim_{x\to 3} \frac{x^2-9}{x-3}$.

c) If $f(x) = \begin{cases} 5x-4 & when\ x<1 \\ 4x^2-3x & when\ x>1 \end{cases}$, find $\lim\limits_{x\to 1} f(x)$.

d) Find the nth derivative of $y = \log(x^2-a^2)$.

Q.No.2. **Answer Any Two Questions** 4×2=8

a) Write a matrix $\begin{bmatrix} 1 & 2 & 4 \\ -2 & 5 & 3 \\ -1 & 6 & 3 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrix.

b) Verify the Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}$.

c) Find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ by using Cayley-Hamilton theorem.

Q.No.3. **Answer Any Two Questions** 4×2=8

a) If $\lim_{x\to a} f(x)$ exist, then prove that it is unique.

b) If $\lim_{x\to a} f(x) = l$ and $\lim_{x\to a} g(x) = m$, then prove that $\lim_{x\to a}[f(x)g(x)] = lm$.

c) Discusses the continuity of $f(x) = \begin{cases} \frac{x^2-25}{x-5} & for\ x \neq 5 \\ 20 & for\ x=5 \end{cases}$.

Q.No.3. **Answer Any Two Questions** 4×2=8

a) State and prove Leibnitz's theorem.

b) Find the nth derivative of $y = \frac{1}{x^2-5x+6}$.

c) Find the nth derivative of $e^{ax}\cos(bx+c)$.

B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE, VIJAYAPUR-586 103

DEPARTMENT OF MATHEMATICS

First Internal Assessment

Sem: III | **Sub:** Ordinary Differential Equations and Real Analysis – I **(DSC)** | **Code:** 21BSC3C3MAT1L

Date: 20 – 01 – 2023 | **Time:** 4:00 PM – 5:00 PM | **Max. Marks: 30**

Q.No.1. Answer Any Three Questions $2\times3=6$

a) Solve $(4x+3y+1)dx+(3x+2y+1)dy=0$

b) Solve $(D^2+25)y=0$.

c) Define sequence. With an example.

d) Show that the series $1^2+2^2+3^2+\cdots+n^2+\cdots$ diverges to $+\infty$.

Q.No.2. Answer Any Three Questions $4\times3=12$

a) State and prove necessary and sufficient condition for the equation to be exact.

b) Solve $xydx-(x^2+2y^2)dy$.

c) With usual notation prove that $\frac{1}{f(D)}e^{ax}V=e^{ax}\frac{1}{f(D+a)}V$, where V is a function of x.

d) Solve $(D^3-3D^2+4D-2)y=e^x+\sin 3x$.

Q.No.3. Answer Any Three Questions $4\times3=12$

a) Prove that every convergent sequence has a unique limit point.

b) Verify the sequence $\{\frac{3n+7}{4n+8}\}$ is monotonic.

c) State and prove p-series test.

d) State and prove Cauchy's general principle of convergence of a series.

B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE, VIJAYAPUR-586 103

DEPARTMENT OF MATHEMATICS

First Internal Assessment

Sem: III | Sub: Ordinary Differential Equations (OEC) | Code: 21BSC303MAT3-A

Date: 20 - 01 - 2023 | Time: 4:00 PM - 5:00 PM | Max. Marks: 30

Q.No.1. Answer Any Three Questions $2\times3=6$

a) Solve $(D^2+4)y=0$.

b) Define Ordinary Differential Equation.

c) Find order and degree of differential equation $\left(1+\left(\frac{dy}{dx}\right)^2\right)^{4/3}=\frac{d^2y}{dx^2}$.

d) What are the three type of equations?

Q.No.2. Answer Any Two Questions $4\times2=8$

a) Solve $(D^3-9D^2+23D-15)y=0$.

b) With usual notation prove that $\frac{1}{f(D^2)}\cos(ax+b)=\frac{\cos(ax+b)}{f(-a^2)}$ provided $f(-a^2)\neq0$.

c) With usual notation prove that $\frac{1}{f(D)}x\,V=x\frac{1}{f(D)}V-\frac{f'(D)}{\{f(D)\}^2}V$, where V is a function of x.

Q.No.3. Answer Any Two Questions $4\times2=8$

a) State and prove Necessary and sufficient condition for exact differential equation.

b) Solve $(3x^2+6xy^2)dx+(6x^2y+4y^3)dy$.

c) Solve $(xy)dx-(x^2+2y^2)dy$.

Q.No.3. Answer Any Two Questions $4\times2=8$

a) Solve $x^2p^2+xyp-6y^2=0$

b) Solve $y=2px+y^2p^3$

c) Solve $p^2-2p\cosh x+1=0$

BLDEA'S

SB ARTS AND KCP SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

BSc-V SEMESTER FIRST INTERNAL TEST 2022-23

PAPER I : Real Analysis **Max. Marks:20**

PART-A

Q.No.1.Answer any five questions. **2X5=10**

1) Prove that every constant function is R-integrable.
2) Define i) Upper Riemann integral.
   ii) Lower Riemann integral.
3) Show that $L(P,f) \leq U(P,f)$.
4) Define i) Beta function.
   ii) Gamma function.
5) Show that $\beta(m,n) = \beta(n,m)$.
6) Show that $\Gamma n + 1 = n\Gamma n$.

PART-B

Q.No.2.Answer any one question. **5x1=5**

1) State and prove Necessary and Sufficient condition of R- integrable.
2) Show that $f(x) = 3x + 1$ is integrable on [1,2] then $\int_1^2 (3x+1)\,dx = \frac{11}{2}$.

PART-C

Q.No.3.Answer any one question. **5x1=5**

1) Show that $\beta(m,n) = \frac{\Gamma m\, \Gamma n}{\Gamma m+n}$, where m,n > 0 .
2) Show that $\int_0^1 \frac{dx}{\sqrt{1-x^n}} \frac{=\sqrt{\pi}\,\Gamma\frac{1}{n}}{n\,\Gamma(\frac{1}{n}+\frac{1}{2})}$.

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SB ARTS AND KCP SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

BSc-V SEMESTER FIRST INTERNAL TEST 2022-23

PAPER II: Numerical Analysis **Max. Marks: 20**

PART-A

Q.No.1. Answer any five questions. **2X5=10**

1) Briefly explain Bisection Method.
2) Briefly explain Iteration Method.
3) Find the cube root of 24 by Newton-Raphson Method with x_0=2.8.
4) Briefly explain Taylor's Series method to solve initial value problems.
5) Define order and degree of difference equation.
6) Find the order of difference equation $y_{n+2} - y_{n+1} + y_n = 2$.

PART-B

Q.No.2.Answer any one question. **5x1=5**

1) Explain Jacobi Iteration Method for system of equations, $a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$.
2) Solve by Gauss-Siedel Iteration Method, $10x + 2y + z = 9$, $2x + 20y - 2z = -44$, $-2x + 3y + 10z = 22$.

PART-C

Q.No.3.Answer any one question. **5x1=5**

1) Solve $\frac{dy}{dx} = x - y^2$ with $y(0) = 1$ by using Taylor's series expansion method and also find $y(0.1)$ correct to 4 decimal places.
2) Form the difference equation by eliminating orbitary constant a and b from the equation $y_n = a2^n + b3^n$.

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SB ARTS AND KCP SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

BSc-V SEMESTER FIRST INTERNAL TEST 2022-23

PAPER II: Numerical Analysis **Max. Marks: 20**

PART-A

Q.No.1. Answer any five questions. **2X5=10**

1) Briefly explain Bisection Method.
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6) Find the order of difference equation $y_{n+2} - y_{n+1} + y_n = 2$.

PART-B

Q.No.2.Answer any one question. **5x1=5**

1) Explain Jacobi Iteration Method for system of equations, $a_1x+b_1y+c_1z=d_1$, $a_2x+b_2y+c_2z=d_2$, $a_3x+b_3y+c_3z=d_3$.
2) Solve by Gauss-Siedel Iteration Method, $10x+2y+z=9$, $2x+20y-2z=-44$, $-2x+3y+10z=22$.

PART-C

Q.No.3.Answer any one question. **5x1=5**

1) Solve $\frac{dy}{dx} = x - y^2$ with $y(0) = 1$ by using Taylor's series expansion method and also find $y(0.1)$ correct to 4 decimal places.
2) Form the difference equation by eliminating orbitary constant a and b from the equation $y_n = a2^n + b3^n$.

B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE, VIJAYAPURA-586 103

DEPARTMENT OF MATHEMATICS

First Internal Assessment

Sem: V **Sub: Mathematics(SEC)**

Date: 21/01/2023 **Max.Marks: 20**

Q.No.1. **Answer Any Five Questions.** **5*2=10**

1) Define Divisibility.
2) Write any five Properties of Divisibility.
3) Define GCD with an example and find (595,252).
4) Find Ø(240) and Ø(64).
5) Find the number of divisors and sum of divisors of 120.
6) Define Euler's Function. With an example.

Q.No.2. **Answer Any Two Questions.** **2*5=10**

1) State and prove Division algorithm theorem.
2) State and prove Euler's theorem.
3) If p is a prime number and r is the positive integer then for Euler's function Ø,
   Prove that $\Phi^k = p^k - p^{k-1}$.

B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE, VIJAYAPUR-586 103

DEPARTMENT OF MATHEMATICS

Second Internal Assessment

Sem : I Sub: Mathematics-I (OEC1) Code: 21BSC101MAT1-A

Date: 06 – 01 – 2023 Time: 4:00 PM – 5:00 PM Max. Marks: 30

Q.No.1. **Answer Any Three Questions.** 2×3=6

a) Define Echelon form of a matrix with an example.

b) Write the condition for consistency and inconsistency of system of linear homogeneous equation.

c) State Lagrange's mean value theorem.

d) State Taylor's Theorem.

Q.No.2. **Answer Any Two Questions.** 4×2=8

a) Reduce the following to its Echelon form, hence find its rank

$$A = \begin{bmatrix} 1 & 2 & -2 & -3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}.$$

b) Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ by using elementary transformation.

c) Solve the system of equations E-transformation $3x + y + 2z = 3$, $2x - 3y - z = -3$, $x + 2y + z = 4$.

Q.No.3. **Answer Any Two Questions.** 4×2=8

a) State and prove Rolle's mean value theorem.

b) Verify Cauchy's mean value theorem for $f(x) = e^x$ and $g(x) = e^{-x}$ in $[a, b]$.

c) Expand $\sin x$ in terms of power x using Maclaurin's Theorem.

Q.No.4. **Answer Any Two Questions.** 4×2=8

a) Find the n^th^ derivative of $x^{n-1} \log x$.

b) If $y = \sin(m \sin^{-1} x)$, then prove that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0.$$

c) If $y = (x^2 - 1)^n$, then prove that

$$(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n + 1)y_n = 0.$$

B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE, VIJAYAPUR-586 103

DEPARTMENT OF MATHEMATICS

Second Internal Assessment

Sem: III **Sub:** Ordinary Differential Equations and Real Analysis – I **(DSC)** **Code:** 21BSC3C3MAT1L

Date: 11- 02 - 2023 **Time:** 12:00 PM – 1:00 PM **Max. Marks:** 30

Q.No.1. Answer Any Three Questions **2×3=6**

a) Solve $p^2 + 2xp - 3x^2 = 0$.

b) State Raabe's test.

c) Test the convergence of the series

$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots\ldots\ldots\ldots$

d) Show that sequence $\{a_n\} = \frac{n+1}{n}$ is converegеs to 1.

Q.No.2. Answer Any Three Questions **4×3=12**

a) Solve $y = 2px + y^2p^3$.

b) Solve $y = -px + x^4p^2$.

c) State and prove the necessary condition for the equation $Pdx + Qdy + Rdz = 0$ to be integrable.

d) Solve $(1 + 2x)^2 \frac{d^2y}{dx^2} - 6(1 + 2x)\frac{dy}{dx} + 16y = 8(1 + x)^2$.

Q.No.3. Answer Any Three Questions **4×3=12**

a) Every monotonically increasing sequence which is bounded above converges to its least upper bound

b) Show that the sequence $\{a_n\}$ defined by by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2a_n}$ converges to 2.

c) State and prove D'Alemberts ratio test.

d) $1+\frac{1}{2}+\frac{1}{2}\frac{3}{4}+\frac{1}{2}\frac{3}{4}\frac{5}{6}+\ldots\ldots\ldots\ldots$test the convergence of the series.

B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE, VIJAYAPUR-586 103

DEPARTMENT OF MATHEMATICS

Second Internal Assessment

Sem: III | **Sub:** Ordinary Differential Equations **(OEC)** | **Code:** 21BSC303MAT3-A

Date: 13- 02 - 2023 | **Time:** 12:00 PM - 1:00 PM | **Max. Marks: 30**

Q.No.1. Answer Any Three Questions $2\times3=6$

a) Solve $(D^2+1)y=1$.

b) Define differential equation. What are the types of differential equation.

c) Define Exact differential equation.

d) Define Singular solution.

Q.No.2. Answer Any Two Questions $4\times2=8$

a) Solve $(D^2+4)y=\sin 2x+e^x$.

b) Solve $(D^2+2D+1)y=x\cos x$.

c) Solve $(D^2-3D+2)y=x^2e^{3x}$.

Q.No.3. Answer Any Two Questions $4\times2=8$

a) Solve $(xy^2+2x^2y^3)dx+(x^2y+x^3y^2)dy=0$.

b) Solve $(3xy-2ay^2)dx+(x^2-2ayx)dy=0$.

c) Solve $(x^2+y^2+2x)dx+2ydy=0$.

Q.No.4. Answer Any Two Questions $4\times2=8$

a) Solve $y=2px+p^4x^2$.

b) Reduce the equation $(px-y)(x-yp)=2p$ to clairuts form by using substitutions $x^2=u$ and $y^2=v$ and then solve.

c) Show that the family of parabolas $y^2=4a(x+a)$ is self orthogonal.

BLDEA's

SB ARTS AND KCP SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

BSc-V SEMESTER SECOND INTERNAL TEST 2022-23

Paper I: Real Analysis

Duration:3hrs Max marks:80

PART-A

I. **Answer any TEN of the following** 10x2=20

1. Define Upper and Lower Riemann Sum.
2. Define norm of a partition.If P={1,1.3,1.5,1.6,1.9,2}be a partition of [1,2] then find norm of P.
3. State Dourboux theorem.
4. Prove that every constant function is R-integrable.
5. Show that i) $U(p,-f) = -L(p,f)$, ii)$L(p,-f) = -U(p,f)$.
6. State Fundamental Theorem of Integral Calculus.
7. Prove that $\left|\int_1^2 \frac{sinx}{x} dx\right| \leq 2$.
8. Define Gamma function and Evaluate $\Gamma 1=1$.
9. Prove the symmetric property of Beta function.
10. Prove that ß(m,n)=2 $\int_0^{\frac{\pi}{2}} sin^{2m-1} cos^{2n-1}\,\theta d\theta$.
11. Evaluate $\int_0^1 x^4(1-x)^3 dx$.
12. Evaluate $\Gamma(\frac{-7}{2})$.

PART-B

II. **Answer any Four of the following** 4x5=20

1. If a function f(x) is bounded on [a,b] then prove that $m(b-a) \leq \int_{\bar{a}}^{b} f(x)dx \leq \int_a^{\bar{\bar{b}}} f(x)dx \leq M(b-a)$.where m and M are infimum and supremum of f on [a,b].
2. If f and g are R-integrable on[a,b] then prove that $f + g$ is also R-integrable.
3. Show that $\frac{1}{3\sqrt{2}} < \int_0^1 \frac{x^2}{\sqrt{1+x^2}} dx < \frac{1}{3}$.
4. prove that every continuous function is R-integrable.
5. Show that $\int_0^\infty \frac{e^{-y^2}}{\sqrt{y}} dy \times \int_0^\infty e^{-y^4} y^2 dy = \frac{\pi}{4\sqrt{2}}$..
6. Prove that ß(m,n)=$\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\, dx$.

PART-C

V. Answer any Four of the following **4x10=40**

1. a) State and prove Necessary and Sufficient condition for integrability of bounded function f(x) in [a,b].

   b) If f(x)=2x+3 then prove that f(x) is R-integrable in [1,2] and prove that $\int_1^2 f(x)dx = 6$.

2. a) State and prove Fundamental theorem of Integral Calculus.

   b) Show that $\frac{\pi}{4} \leq \int_0^{\frac{\pi}{4}} secxdx \leq \frac{\pi}{2\sqrt{2}}$.

3. a) State and prove First mean value theorem.

   b) Show that the function 'f' defined by $f(x) = \frac{1}{2^n}$ when $\frac{1}{2^{n+1}} < x \leq \frac{1}{2^n}$

f(x)=0 is integrable

on [0,1] and evaluate $\int_0^1 f(x)dx.$ (where n=0,1,2,3........n-1).

4. a) State and Prove Duplication formula and hence reduce

   $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$

   b) Express $\int_0^1 \frac{x^2}{\sqrt{1-x^5}} dx$ *in terms of Beta and Gamma Function.*

5. a) Show that ß(m,n)= $\frac{\Gamma m\, \Gamma n}{\Gamma m+n}$.

   b) *Evaluate* $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\, d\theta$.

BLDEA's

SB ARTS AND KCP SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

BSc-V SEMESTER SECOND INTERNAL TEST 2022-23

Paper II : Numerical Analysis

Duration:3hrs **Max marks:80**

PART-A

I. Answer any TEN of the following. **10x2=20**

1. Find the root of equation $x^3 - 4x - 9 = 0$ using Bisection method in 2 iterations.
2. Find the root of equation $e^x - 3x = 0$ using Iteration method.
3. Find the iterative formula for $\sqrt{N}$.
4. Evaluate $\Delta logx$.
5. Define forward difference and write first forward difference.
6. Define backward difference and write first backward difference.
7. Explain the Picard's method to solve the initial value problem of first order ordinary differential equation.
8. Write a formula of Trapezoidal rule.
9. Define general and particular solution of a difference equation.
10. Find the order of the following difference equation $y_{n+2} - 4y_{n+1} + y_n = 6$.
11. Find the relation between the following difference equation $\Delta Y_{n+1} + Y_n = 2$ and $\Delta Y_{n+1} + \Delta^2 Y_{n-1} = 1$.
12. Form the difference equation from the eliminating 'a' from $Y_n = a5^n$.

PART-B

II. Answer any Four of the following. **4x5=20**

1. Derive Newton-Raphson method.
2. Derive Gauss-Seidal iteration method.
3. Construct forward difference table , Find $\Delta^4 5$, $\Delta^2 20, \Delta 25$.

x	5	10	15	20	25	30
y	9962	9848	9659	9397	9063	8660

4. Derive 'General Quadrature Formula' for equidistant ordinates.
5. Apply Euler's method to solve the initial value problem $\frac{dy}{dx} = x + y$ with $y(0) = 0$ and h=0.2 and also find $y(1)$.
6. Form the difference equation corresponding to the family of curve $y = ax + bx^2$.

PART-C

III. Answer any Four of the following. **4x10=40**

1. a) If f(x) be polynomial of n^{th} degree in x, then $\Delta^n f(x)$ is constant and $\Delta^{n+1} f(x) = 0$.

   b) Express $f(x) = 2x^3 - 3x^2 + 3x - 10$ in factorial notation and also find it's successive differences.

2. a) Derive Jacobi iteration method.

   b) Use Bisection method for $cosx - xe^x = 0$ for 6 iterations.

3. a) Find the root of equation $x^3 - x - 2 = 0$ in [1,2] by Newton Raphson method.

   b) Explain the Euler's method to solve the initial value problem of first order ordinary differential equation.

4. a) Using Taylor's series method obtained approximation value of y at $x = 0.2$ for the differential equation $\frac{dy}{dx} = 2y + 3e^x$ with $y(0) = 0$.

   b) Using Euler's modified method find $y(0.1)$ given $\frac{dy}{dx} = x - y^2$ with $y(0) = 1$ taking $h = 0.1$.

5. a) Solve $[E^3 - 5E^2 + 3E + 9] = 2^n + 3^n$.

   b) Solve $Y_{n+2} - 4Y_n = n - 1$.

B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE, VIJAYAPURA-586 103

DEPARTMENT OF MATHEMATICS

Second Internal Assessment

Sem: V Sub: Mathematics(SEC)

Date: 14/02/2023 Max.Marks: 40

Q.No.1. Answer Any Five Questions. 5*2=10

1) Define Divisibility.
2) Define LCM with an example.
3) Define Proper and Improper divisor.
4) Define Bracket function with an example.
5) Find the highest power of 3 in 1000!.
6) Show that $6^{52} - 1$ is divisible by 53.

Q.No.2. Answer Any Three Questions. 3*5=15

1) State and prove Division algorithm theorem.
2) Let a & b be two positive integers, if $d(a,b)$ and $m = [a,b]$ then Prove that $(a,b)[a,b] = ab$.
3) Find (256,1166) & Express GCD as a linear combination of 256 and 1166.
4) Find [8,12,15,20,25].

Q.No.3. Answer Any Three Questions. 3*5=15

1) If $a > 0, b > 0$ $(a,b) = 1$ then show that $\o(n) = \o(a).\o(b)$
2) State and prove Euler's theorem.
3) For any n€N, ø(n) = $n\prod(1-\frac{1}{p})$(this is called product formula for ø(n)).
4) State and prove Fermat's theorem.

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B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE, VIJAYAPUR-586 103

DEPARTMENT OF MATHEMATICS

First Internal Assessment

Sem: II | **Sub:** Algebra-II and Calculus-II **(DSC1)** | **Code:** 21BSC2C2MAT2L

Date: 12-07-2023 | **Time:** 01:30PM-02:30PM | **Max. Marks:** 30

Q.No.1. Answer Any Three Questions $2\times3=6$

a) Define finite set with an example.

b) Define group with an example.

c) Find $\frac{dy}{dx}$ using partial differentiation $x^y = y^x$.

d) Evaluate $\int_0^1\int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$.

Q.No.2. Answer Any Three Questions $4\times3=12$

a) Prove that a subset of finite set is finite.

b) Prove that the union of countable collection of countable set is countable.

c) If a and b are any two elements of a group (G,*) then prove that

$$(a*b)^{-1} = b^{-1}*a^{-1} \text{ for all } a,b \in G.$$

d) Show that the set $Z_6 = \{0,1,2,3,4,5\}$ is an abelian group with respect to $\oplus_6$.

Q.No.3. Answer Any Three Questions $4\times3=12$

a) State and prove Euler's theorem for homogeneous functions.

b) If $u = f(x,y)$ where $x = r\cos\theta, y = r\sin\theta$, then prove that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2.$$

c) Evaluate $\int_C (x+y)dx + (y-x)dy$

i) Along the parabola $y^2 = x$ from (1,1) to (4,2).

ii) Along the line joining (1,1) to (4,2).

d) Find the value of double integral $\iint_R x^2dydx$ where R is the $2D$ region bounded by the curve $y = x$ and $y = x^2$.

B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE, VIJAYAPUR-586 103

DEPARTMENT OF MATHEMATICS

First Internal Assessment

Sem: II **Sub:** Mathematics-I (OEC1) **Code:** 21BSC202MAT2-A

Date: 12-07-2023 **Time:** 10:00AM - 11:00AM **Max. Marks:** 30

Q.No.1. Answer Any Three Questions $2\times3=6$

a) Define semi group with an example.

b) Define monoid with an example.

c) Using partial differentiation find $\frac{dy}{dx}$ if $y^x = x$.

d) Evaluate line integral $\int_C (x^2 - y)\,dx + (y^2 + x)dy$ where C is curve given by $x = t, y = t^2 + 1$ and $0 \leq t \leq 1$.

Q.No.2. Answer Any Two Questions $4\times2=8$

a) Prove that the inverse of an element in a group G is unique.

b) Show that the set $Z_4 = \{0,1,2,3\}$ is an abelian group with respect to $\oplus_4$.

c) In a set Q of rational numbers the binary operation $*$ is defined as $a * b = \frac{ab}{7}$ for all $a, b \in Q$.

Q.No.3. Answer Any Two Questions $4\times2=8$

a) If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$, then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

b) State and prove Euler's theorem for homogeneous functions.

c) If $u = f(y - z, z - x, x - y)$, then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.

Q.No.3. Answer Any Two Questions $4\times2=8$

a) Evaluate $\int_C (3x + y)\,dx + (2y - x)dy$

i) Along the curve $y = x^2 + 1$ from $(0,1)$ to $(3,10)$.

ii) Along the line joining $(0,1)$ to $(3,10)$.

b) Evaluate $\iint_R xydydx$ over the positive codrent of circle $x^2 + y^2 = a^2$.

c) Evaluate $\int_0^1 \int_x^1 (x^2 + y^2)dydx$ by changing the order of integration.

B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE, VIJAYAPUR-586 103

DEPARTMENT OF MATHEMATICS

First Internal Assessment

Sem: IV | Sub: – I (DSC) | Code: 21BSC4C4MAT2L

Date: 10– 07 – 2023 | Time: 1:30 PM – 2:30 PM | Max. Marks: 30

Q.No.1. Answer Any Three Questions $2\times3=6$

a) Form the partial differential equation by eliminating arbitrary function $f(x^2+y^2+z^2, x+y+z)=0.$

b) Solve $(D^2-DD'-6D'^2)z=0.$

c) Define Fourier series of functions of 2π and $2l.$

d) Find $L[\cosh at].$

Q.No.2. Answer Any Three Questions $4\times3=12$

a) Solve $x(z^2-y^2)p+y(x^2-z^2)q=z(y^2-x^2).$

b) State and prove Lagrange's linear equation.

c) Solve $4(r-s)+t=16\log(x+2y).$

d) Solve $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=\cos mx\cos ny.$

Q.No.3. Answer Any Three Questions $4\times3=12$

a) Find the Fourier series in the interval $(-\pi,\pi)$ for the function $f(x)=|x|$ and deduce that

$$\frac{\pi^2}{8}=1+\frac{1}{3^2}+\frac{1}{5^2}+\cdots=\sum_{n=1}^{\infty}\frac{1}{2n-1)^2}.$$

b) Obtain the Fourier series for the function

$$f(x)=\begin{cases}-1\ if\ -1<x<0\\ 2x\ if\ \ 0<x<1\end{cases}.$$

c) State and prove periodic function.

d) Find $L[\cos5t\cdot\cos3t].$

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BLDEA'S

SB ARTS AND KCP SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

BSc-VI SEMESTER FIRST INTERNAL TEST 2022-23

PAPER I:Complex Analysis and Ring Theory **Max.Marks:20**

PART-A

Q.No.1.Answer any five questions. **2X5=10**

1) Prove that an analytic function with constant real part is constant .
2) Show that $f(z) = xy + iy$ is not analytic.
3) Prove that $\frac{1}{2} log(x^2 + y^2)$ is harmonic.
4) In a ring $(R, +,*)$, prove that $i)\ a(b-c) = ab - ac$
   ii) $(b-c)a = ba - ca$
5) Define Integral domain and division ring with examples.
6) Define Left ideal and Right ideal.

PART-B

Q.No.2.Answer any one question. **5x1=5**

1) State and prove necessary condition for a function $f(z)$ to be analytic.
2) If $v= e^x(xsiny + ycosy)$ find $f(z)$ in terms of z by using Milne-thomson method.

PART-C

Q.No.3.Answer any one question. **5x1=5**

1) Prove that a non empty subset S of a ring is a subring of R iff $a \in S, b \in S$ then
   a) $a - b \in S$
   b) $ab \in S$
2) Let $f: R \to R'$ be a homomorphism from the ring R into R' then
   i)$f(0) = 0'$ where 0 and 0' are the zero's of R and R' respectively.
   ii)$f(-a) = -f(a) \quad \forall a \in R.$

B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE VIJAYAPUR-586 103

DEPARTMENT OF MATHEMATICS

First Internal Test 2022-23

Paper II : Differential Equations

Max. Marks: 20

PART-A

Q.No.1. Answer Any Five Questions. **2x5=10**

a) Define ordinary and singular point of differential equation.

b) Show that $x = 0$ is an ordinary point of $(x^2 - 1)y'' + xy' - y = 0$.

c) Solve $\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$.

d) Solve $(D^3 - 2D^2D' - DD'^2 + 2D'^3)z = 0$.

e) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.

f) Solve $(D^2 - 4DD' + 4D'^2)z = 0$.

PART-B

Q.No.2. Answer Any One Question **5x1=5**

a) Solve $y' - y = 0$ about $x = 0$ by power series.

b) Solve $4xy'' + 2y' + y = 0$ by Frobenius method.

PART-C

Q.No.3. Answer Any One Question. **5x1=5**

a) Solve $\frac{dx}{dt} = x + y$ and $\frac{dy}{dt} = 4x - 2y$.

b) Solve $px + qy = pq$ by Charpit's Method.

B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE, VIJAYAPUR-586 103

DEPARTMENT OF MATHEMATICS

First Internal Assessment

Sem: VI   Sub: GRAPH THEORY

Date: 13 – 07 – 2023   Time: 12:00 PM – 1:00 PM   Max. Marks: 20

Q.No.1. **Answer Any Five Questions** **2×5=10**

a) Define pseudo graph give an example.

b) Define multiple edges give an example.

c) Define isolated vertex and pendent vertex give an example.

d) Define subgraph give an example.

e) Define induced subgraph and write the types.

f) Find the total number of subgraphs and spanning subgraphs in K_4.

Q.No.2. **Answer Any One Question** **5×1=5**

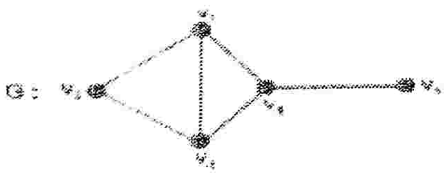
a) State and prove Handshaking theorem.

b) i) Define complete graph give an example.

ii)Find 0-regular and 2-regular graph with 4 vertices.

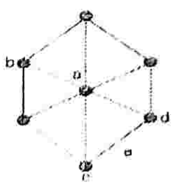
Q.No.3. **Answer Any one Question** **5×1=5**

a) Define vertex disjoint and edge disjoint subgraphs and construct two edge disjoint and vertex disjoint subgraph of a graph G shown below.



b) For a graph G shown below ,draw the subgraphs.

i) G – e ii) G – a iii) G – b iv) G – d



B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE, VIJAYAPUR-586 103

DEPARTMENT OF MATHEMATICS

Second Internal Assessment

Sem: II | Sub: Algebra-II and Calculus-II (DSC1) | Code: 21BSC2C2MAT2L

Date: 24-08-2023 | Time: 01:30PM-02:30PM | Max. Marks: 30

Q.No.1. Answer Any Three Questions $2\times3=6$

a) Define open set with an example.

b) Define cyclic group with an example.

c) If $u = x^2 + y^2$ and $v = 2xy$, then find $\frac{\partial(u,v)}{\partial(x,y)}$.

d) Evaluate $\int_1^2 \int_0^1 \int_{-1}^1 (x^2 + y^2 + z^2)\, dx\, dy\, dz$.

Q.No.2. Answer Any Three Questions $4\times3=12$

a) Prove that $i)\ lub(-A) = -glb(A)$

$ii) glb(-A) = -lub(A)$.

b) State and prove Archimedeans property of R.

c) Prove that a non empty subset H of a group $(G,*)$ is a subgroup of G if and only if $a * b^{-1} \in H$.

d) State and prove Lagrange's theorem for finite group.

Q.No.3. Answer Any Three Questions $4\times3=12$

a) If $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$, then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.

b) State and prove Taylor's theorem for function of two variables.

c) Find the area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration.

d) State and prove Leibnitz's theorem for differentiation under integral sign.

B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE, VIJAYAPUR-586 103

DEPARTMENT OF MATHEMATICS

Second Internal Assessment

Sem: II **Sub:** Mathematics-I **(OEC1)** **Code:** 21BSC202MAT2-A

Date: 24-08-2023 **Time:** 10:00AM – 11:00AM **Max. Marks: 30**

Q.No.1. Answer Any Three Questions **2×3=6**

a) Define cyclic group with an example.

b) State Lagrange's theorem for finite group.

c) If $x = r\cos\theta, y = r\sin\theta$, then find $\frac{\partial(x,y)}{\partial(u,v)}$.

d) Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2yz \; dx\,dy\,dz$.

Q.No.2. Answer Any Two Questions **4×2=8**

a) Prove that a non empty subset H of a group $(G,*)$ is a subgroup of G if and only if $a * b^{-1} \in H$.

b) Prove that every subgroup of cyclic group is cyclic.

c) State and prove Fermat Theorem.

Q.No.3. Answer Any Two Questions **4×2=8**

a) Expand $\sin x \cos y$ by Maclaurin,s series upto 3rd terms.

b) If $x = \frac{u^2}{v}, y = \frac{v^2}{u}$, then find $\frac{\partial(u,v)}{\partial(x,y)}$.

c) With usual notation prove that $\frac{\partial(u,v)}{\partial(x,y)}\frac{\partial(x,y)}{\partial(u,v)} = 1$.

Q.No.4. Answer Any Two Questions **4×2=8**

a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2\sqrt{x^2-y^2}\, dy\, dx$ by transforming to the polar co-ordinates.

b) Find the area enclosed by the curve cardioid $r = a(1+cos\theta)$ between $\theta = 0$ to $\theta = \pi$.

c) Using differentiation under integral sign evaluate $\int_0^1 \frac{x^a-1}{logx}\, dx$.

B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE, VIJAYAPUR-586 103

DEPARTMENT OF MATHEMATICS

Second Internal Assessment

Sem: IV Sub: – Partial Differential Equation and Integral Transform -I **(DSC)** Code:21BSC4CMAT2L

Date: 22– 08 – 2023 Time: 1:30 PM – 2:30 PM Max. Marks: 30

Q.No.1. Answer Any Three Questions **2×3=6**

a) Find the singular integral of $z = px + qy + pq$

b) Classify $xyr - (x^2 - y^2)s - xyt + yp - xq = 2(x^2 - y^2)$.

c) Find the Laplace transform for $L[t.e^{at}]$.

d) Find finite Fourier sine transform of $f(x) = x$ in $(0, l)$

Q.No.2. Answer Any Three Questions **4×3=12**

a) Solve$p^3 + q^3 = 3pqz$.

b) Solve by Charpit's method $(p^2 + q^2)y = qz$.

c) Obtain the solution of Laplace equation $u_{xx} + u_{yy} = 0$ by the method of separations variable.

d) Reduce $r + 2xs + x^2t = 0$ to canonical form.

Q.No.3. Answer Any Three Questions **4×3=12**

a) Find the Laplace transform of dirac delta function i.e. $L[\delta(t-a)]$.

b) Express the following function in terms of Heaviside function and find their Lapalce transforms of

$$f(t) = \begin{cases} 5 & 0 \le t \le 4 \\ t & t > 4 \end{cases}.$$

c) Find half-range Fourier sine and cosine series of

$$f(x) = \begin{cases} x & 0 < x < \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} < x < \pi \end{cases}.$$

d) Find finite Fourier sine and cosine transformations of the function f(x)=e^{ax} in the interval $(0, l)$.

BLDEA'S

SB ARTS AND KCP SCIENCE COLLEGE, VIJAYAPUR

DEPARTMENT OF MATHEMATICS

BSc-VI SEMESTER SECOND INTERNAL TEST 2022-23

PAPER I: Complex Analysis and Ring Theory Max.Marks:20

PART-A

Q.No.1.Answer any five questions. 2X5=10

1) Evaluate $\int_c (\bar{z})^2\ dz$ around the circle $|z| = 1$.
2) Define closed curve and contour.
3) State Morere's theorem.
4) Expand $f(z) = e^z$ in the form of Taylor's series about $z = 0$.
5) Find the residue of $f(z) = \frac{e^z}{(z)(z-2)}$.
6) Define i) Pole
   ii) Removable Singularity.

PART-B

Q.No.2.Answer any one question. 5x1=5

1) State and prove Cauchy's integral formula.
2) Show that $\int_c \frac{3z-1}{(z+1)(z-3)}\ dz = 6\pi$ where c: $|z| = 4$.

PART-C

Q.No.3.Answer any one question. 5x1=5

1) State and prove Cauchy's residue theorem.
2) Prove that $\int_0^{2\pi} \frac{d\theta}{5+4cos\theta} = \frac{2\pi}{3}$.

B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE, VIJAYAPUR-586 103

DEPARTMENT OF MATHEMATICS 2022-23

Second Internal Assessment

Sem: VI   Sub: Differential Equations   Max. Marks: 20

Q.No.1. Answer Any Five Questions 2×5=10

a) Write Legendre's polynomial of first kind.

b) Show that $P_n(1) = 1$.

c) Form a partial differential equation by eliminating a and b from $z = ax + by + ab$.

d) Form a partial differential equation by eliminating a and b from $z = (x-a)^2 + (y-b)^2$.

e) Solve pq=k.

f) Solve $(D^2 + D'^2)z = x^2y^2$.

Q.No.2. Answer Any One Question 5×1=5

a) Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = 0$, if $m \neq n$.

b) State and prove Rodrigues formula.

Q.No.3. Answer Any One Question 5×1=5

a) Find the condition for integrability of the equation $Pdx + Qdy + Rdz = 0$ where P, Q, R are functions of x, y and z.

b) Find the singular integral of $z = px + qy + pq$.

B.L.D.E.A's

S.B. ARTS AND K.C.P. SCIENCE COLLEGE, VIJAYAPUR-586 103

DEPARTMENT OF MATHEMATICS

Second Internal Assessment

Sem: VI Sub: GRAPH THEORY

Date: 25- 08- 2023 Time: 12:00 PM – 1:00 PM Max. Marks: 20

Q.No.1. **Answer Any Five Questions** 2×5=10

a) Define line graph give an example.

b) Define maximum degree an minimum degree give an example.

c) Define compliment of a graph give an example.

d) Define walk give an examples.

e) Define Bipartite graph give an example.

f) Prove that C_6 is bipartite graph.

Q.No.2. **Answer Any One Question** 5×1=5

a) State and prove Handshaking theorem.

b) Prove that the number of vertices of odd degree is even.

Q.No.3. **Answer Any one Question** 5×1=5

a) For any graph G with six vertices G or $\bar{G}$ contains a triangle.

b) Prove that A graph is bipartite iff it contains no odd cycles.