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S. B. ARTS & K. C. P. SCIENCE COLLEGE, VIJAYAPUR

I^{st} INTERNAL TEST 2023 -2024

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3a] Schrodinger time Independent wave equation

W.K.T $\psi(x,t) = A e^{-2\pi i(\nu t - \frac{x}{\lambda})}$

$\psi(x,t) = A e^{-\frac{2\pi i}{h}(Et - Px)}$ —①

$\psi(x,t) = A e^{-(\frac{2\pi i}{h}E)t} \cdot e^{(\frac{2\pi i}{h}P)x}$ —②

$\psi(x,t) = \psi_0 e^{-(2\pi i \frac{E}{h})t}$

$\therefore \psi_0 = A e^{(\frac{2\pi i}{h}P)x}$

$\psi(x,t) = \psi_0 e^{-(\frac{2\pi i E}{h})t}$ —③

Eqⁿ ③ diff w.r.to 't'

$$\frac{\partial \psi(x,t)}{\partial t} = \psi_0 e^{-(\frac{2\pi i E}{h})t} \cdot \left(-\frac{2\pi i E}{h}\right)$$

$$\frac{\partial \psi}{\partial t} = \psi_0\left(-\frac{2\pi i E}{h}\right) \quad \Big| \therefore \psi_0 e^{-(\frac{2\pi i E}{h})t} = \psi$$

$$\frac{\partial \psi}{\partial t} = -\psi \frac{2\pi i E}{h} \quad —④$$

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Eqⁿ ③ diff w.r.to 'x'

$$\frac{\partial \psi}{\partial x} = \frac{\partial \psi_0}{\partial x} e^{-(\frac{2\pi i E}{h})t}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2 \psi_0}{\partial x^2} e^{-(\frac{2\pi i E}{h})t} \quad —⑤$$

consider Schrodinger time ~~de~~ independent eqⁿ

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi$$

eqⁿ ④ & ⑤

$$i\hbar\left[-\psi \frac{2\pi i E}{h}\right] = \left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2 \psi_0}{\partial x^2} e^{-(\frac{2\pi i E}{h})t}\right)\right] + V\psi$$

$$E\psi = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi$$

$$\therefore \psi_0 e^{-(\frac{2\pi i E}{h})t} = \psi$$

In 1-D $\boxed{E\psi = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi}$

In 3-D $\boxed{E\psi = -\frac{\hbar^2}{2m}\nabla^2 \psi + V\psi}$

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3b) Statement :- consider two Eigen functions ψ_m & ψ_n these two f^ns are said to be orthonormalized

i.e. $\int_{-\infty}^{\infty} \psi_m^* \psi_m = \delta_{mn}$ where $\delta_{mn} = \begin{cases} 0 & m \neq n \\ 1 & m = n \end{cases}$

proof :- consider schrodinger time independent Eq^n & is given by

$$E\psi = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi \quad \text{--- ①}$$

Eq^n① can be written as ψ_m & ψ_n of the wave f^n

$$E_m\psi_m = \frac{-\hbar^2}{2m}\frac{\partial^2\psi_m^*}{\partial x^2} + V\psi_m^*$$

$$E_m\psi_m = \frac{-\hbar^2}{2m}\frac{\partial^2\psi_m}{\partial x^2} + V\psi_m \quad \text{--- ①}$$

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$$E_n\psi_n = \frac{-\hbar^2}{2m}\frac{\partial^2\psi_n}{\partial x^2} + V\psi_n \quad \text{--- ②} \quad \times \psi_m^*$$

It's complex conjugate the above Eq^ns.

$$E_m\psi_m^* = \frac{-\hbar^2}{2m}\frac{\partial^2\psi_m^*}{\partial x^2} + V\psi_m^* \quad \text{--- ③} \quad \times \psi_n$$

$$E_n\psi_n^* = \frac{-\hbar^2}{2m}\frac{\partial^2\psi_n^*}{\partial x^2} + V\psi_n^* \quad \text{--- ④}$$

from Eq^n ② multiply by ψ_m^* & Eq^n ③ multiply by ψ_n by left then substract

$$\psi_n E_m\psi_m^* - \psi_m^* E_n\psi_n = \left[-\frac{\hbar^2}{2m}\psi_n\frac{\partial^2\psi_m^*}{\partial x^2} + \psi_n V\psi_m^*\right] - \left[\frac{-\hbar^2}{2m}\psi_m^*\frac{\partial^2\psi_n}{\partial x^2} + \psi_m^* V\psi_n\right]$$

$$(E_m - E_n)\psi_n\psi_m^* = -\frac{\hbar^2}{2m}\psi_n\frac{\partial^2\psi_m^*}{\partial x^2} + \cancel{\psi_n V\psi_m^*} + \frac{\hbar^2}{2m}\psi_m^*\frac{\partial^2\psi_n}{\partial x^2} - \cancel{\psi_m^* V\psi_n}$$

$$(E_m - E_n)\psi_n\psi_m^* = \cancel{-}\frac{\hbar^2}{2m}\left[\psi_n\frac{\partial^2\psi_m^*}{\partial x^2} - \psi_m^*\frac{\partial^2\psi_n}{\partial x^2}\right]$$

$$\frac{2m}{\hbar^2}[E_m - E_n]\psi_n\psi_m^* = -\left[\psi_n\frac{\partial^2\psi_m^*}{\partial x^2} - \psi_m^*\frac{\partial^2\psi_n}{\partial x^2}\right]$$

$$\frac{2m}{\hbar^2}[E_m - E_n]\psi_n\psi_m^* = \left[\psi_m^*\frac{\partial^2\psi_n}{\partial x^2} - \psi_n\frac{\partial^2\psi_m^*}{\partial x^2}\right]$$

Integration on b.s betn the limit $+\infty$ to $-\infty$

$$\frac{2m}{\hbar^2}(E_m - E_n)\int_{\infty}^{\infty} \psi_n \psi_m^* \, dx = \int_{-\infty}^{\infty}\left(\psi_m^* \frac{\partial^2 \psi_n}{\partial x^2} - \psi_n \frac{\partial^2 \psi_m^*}{\partial x^2}\right) dx$$

$$\frac{2m}{\hbar^2}(E_m - E_n)\int_{-\infty}^{\infty} \psi_n \psi_m^* \, dx = \left[\psi_m^* \frac{\partial \psi_n}{\partial x} - \psi_n \frac{\partial \psi_m^*}{\partial x}\right]$$

$x \to \pm\infty$ ψ_m^* $\partial\psi_n/\partial x$, $\partial\psi_m/\partial x$ vanishes.

$$\frac{2m}{\hbar^2}(E_m - E_n)\int_{-\infty}^{\infty} \psi_n \psi_m^* \, dx = 0$$

$$\frac{2m}{\hbar^2}(E_m - E_n) \neq 0.$$

$$\int_{-\infty}^{\infty} \psi_m^* \psi_n \, dx = 0$$

$m = n$

i) $\int_{-\infty}^{\infty} \psi_m^* \psi_n \, dx = 1$

$m \neq n$

ii) $\int_{-\infty}^{\infty} \psi_m^* \psi_n \, dx = 0$

1] a] Schrodinger time dependent eq^n $\psi(x,t) = Ae^{i(Kx - \omega t)}$

$K = \frac{2\pi}{\lambda}$ $\omega = 2\pi\nu$

Let $\psi(x,t) = Ae^{-2\pi i(\nu t - x/\lambda)}$ —①

$E = h\nu$, $\lambda = h/p$

$$\psi(x,t) = Ae^{\frac{-2\pi i}{h}(Et - px)} \quad —②$$

Eq^n ② diff w.r.to t.

$$\frac{\psi(x,t)}{\partial t} = Ae^{-\left(\frac{2\pi i}{h}E\right)\cdot t}\left(-\frac{2\pi i E}{h}\right)$$

$$\frac{\partial \psi}{\partial t} = \psi\left(-\frac{2\pi i E}{h}\right) = -\frac{2\pi i E}{h}\psi \quad —③$$

Eq^n ② diff w.r.to x

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$$\frac{\partial \psi}{\partial x} = Ae^{-\left(\frac{2\pi i}{h}p\right)x} \cdot \left(-\frac{2\pi i p}{h}\right)$$

$$\frac{\partial^2 \psi}{\partial x^2} = Ae^{-\left(\frac{2\pi i}{h}p\right)x}\left(-\frac{2\pi i p}{h}\right)^2$$

$$\frac{\partial^2 \psi}{\partial x^2} = \psi\left(-\frac{2^2\pi^2 i^2 p^2}{h^2}\right)$$

$$\frac{\partial^2 \psi}{\partial x^2} = \pm\frac{4\pi^2 p^2}{h^2}\psi \quad —④$$

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Total Energy is equal to the sum of K.E & potential Energy

$$E = \frac{+p^2}{2m} + V(x,t) \quad \text{--- (5)}$$

Eqⁿ (5) multiply by "ψ"

$$E\psi = \frac{p^2}{2m}\psi + V\psi \quad \text{--- (7)}$$

from Eqⁿ (3) & (4) substitute in Eqⁿ (7)

$$\frac{-h}{2\pi i}\frac{\partial\psi}{\partial t} = \frac{\cancel{p^2}}{2m}\cdot\left(-\frac{h^2}{4\pi^2}\frac{\partial^2\psi}{\partial x^2}\right) + V\psi$$

$$\frac{-h}{2\pi i}\frac{\partial\psi}{\partial t} = \frac{h^2}{8\pi^2 m}\frac{\partial^2\psi}{\partial x^2} + V\psi$$

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi}$$

It is S. time dependent 1-D Eqⁿ.

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi + V\psi}$$ It is 3-D Eqⁿ S.D.E.

b] i] $[z, P_z]$

$z = z \qquad P_z = -i\hbar \frac{\partial}{\partial z}$

consider $[z, P_z] f(x)$

$[z P_z] f(x) = [z P_z - P_z z] f(x)$

$= [z(-i\hbar \frac{\partial}{\partial z}) - (-i\hbar \frac{\partial}{\partial z}) z] f(x)$

$= [-z i\hbar \frac{\partial}{\partial z} + i\hbar \frac{\partial z}{\partial z}] f$

$= [-z i\hbar \frac{\partial f}{\partial z} + i\hbar \frac{\partial z}{\partial z}(f)]$

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$= -z i\hbar \cancel{\frac{\partial f}{\partial z}} + i\hbar (f) + i\hbar z \cancel{\frac{\partial f}{\partial z}}$

$[z P_z] \cancel{f} = i\hbar \cancel{f}$

$\boxed{[z, P_z] = i\hbar}$

ii] $[x, P_x^n]$

W.K.T $[x P_x]^n$

If $n = 1 \qquad [x P_x^1]$

Let $= [x P_x - P_x x] f(x)$

$= [x(-i\hbar \frac{\partial}{\partial x}) - (-i\hbar \frac{\partial}{\partial x}) x] f(x)$

$= -x i\hbar \cancel{\frac{\partial f(x)}{\partial x}} + i\hbar x \cancel{\frac{\partial f(x)}{\partial x}} + i\hbar f(x)$

$[x P_x] \cancel{f(x)} = i\hbar \cancel{f(x)}$

$[x P_x] = i\hbar$

if $n=1 \quad [x P_x^n] \Rightarrow [x P_x^1] = i\hbar$

lly if $n=2 \quad [x P_x^n] \Rightarrow [x P_x^2] = 2i\hbar P_x$

if $n=3 \quad [x P_x^n] \Rightarrow [x P_x^3] = 3i\hbar P_x^2$

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if $n=4$ $\quad [x P_x^n] = [x P_x^4] = 4 i\hbar P_x^3$

$\vdots$

In general

If $n=n$ $\quad [x P_x^n] = n i\hbar P_x^{n-1}$

$$\boxed{[x P_x^n] = n i\hbar P_x^{n-1}}$$

$$\begin{aligned}
\text{lly} \quad [x, P_x^2] &\Rightarrow [x, P_x P_x] \\
&= P_x(x P_x) + P_x(x P_x) \\
&= P_x(i\hbar) + P_x(i\hbar) \\
&= 2 i\hbar P_x
\end{aligned}$$

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Ist INTERNAL TEST 2022 -2023

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Subject/ವಿಷಯ : AMMP Semester/ಸೆಮಿಸ್ಟರ್ : IVth Exam Seat No / ಪರೀಕ್ಷೆ ಕ್ರಮಾಂಕ :

Invigilator's Signature / ಕೊಠಡಿ ಮೇಲ್ವಿಚಾರಕರ ಸಹಿ Signature of Evaluator / ಪರೀಕ್ಷಕರ ಸಹಿ 04/04 Marks obtained/ ಪಡೆದ ಅಂಕಗಳು 19/20

1.

a) Diffusion or Heat flow equation:

→ the heat flow equation is,

$$\nabla^2 u = \frac{1}{\alpha^2}\frac{\partial u}{\partial t} \quad \text{——— (1)}$$

u = temprecture

∇^2 = rectangular plante.

α = constant

there fs flow of the heat is,

$$u = F(x,y,z)\,T(t) \quad \text{——— (2)}$$

from eqn (1) and (2),

$$\nabla^2(TF) = \frac{1}{\alpha^2}\frac{\partial(TF)}{\partial t}$$

$$T\nabla^2 F = \frac{1}{\alpha^2} F\frac{\partial T}{\partial t} \quad \text{——— (3)}$$

the eqn (3) divided by (FT)

$$\frac{1}{F}\nabla^2 F = \frac{1}{\alpha^2 T}\frac{\partial T}{\partial t}$$

then, the constant,

$$\frac{1}{F}\nabla^2 F = \frac{1}{\alpha^2 T}\frac{\partial T}{\partial t} = -k^2$$

then separate the equation is

$$\frac{1}{F}\nabla^2 F = -k^2 \Longrightarrow \nabla^2 F + k^2 F = 0$$

$$\frac{1}{\alpha^2 T}\frac{\partial T}{\partial t} = -k^2 \Longrightarrow \frac{\partial T}{\partial t} + k^2\alpha^2 t = 0.$$

(then, Laplace eqn.

$$u = e^{-ny}\sin kx$$

$$u = e^{-n}\quad \sin kx)$$

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$$F = \begin{Bmatrix} \sin kx \\ \cos kx \end{Bmatrix} \qquad T = \begin{Bmatrix} e^{-ky} \\ e^{-ky} \end{Bmatrix}$$

(at $x = 0\ l$ at $y = 0$

the $\sin kx = n\pi$)

$k = \frac{n\pi}{l}$

$$\boxed{k = \frac{n\pi}{l}}$$

$$\boxed{y = e^{-\frac{\alpha^2 \cancel{k^2} y \pi^2}{}} \sin kx}$$

$$\boxed{y = e^{-\frac{\alpha^2 n^2 \pi^2 y}{l^2}} \sin kx}$$

(2) Wave equation :

the wave equation is,

$$\frac{\partial^2 x}{\partial y^2} = \frac{1}{v^2} \frac{\partial^2 x}{\partial t^2} \quad \text{——— (1)}$$

then,

(then ~~$y = x$~~) $x = y(y) \cdot t(t)$

$x = yt$

$$\frac{\partial^2 yt}{\partial y^2} = \frac{1}{v^2} \frac{\partial^2 (yt)}{\partial t^2} \quad \text{——— (2)}$$

$$t \frac{\partial^2 y}{\partial y^2} = \frac{1}{v^2} y \frac{\partial^2 t}{\partial t^2} \quad \text{——— (3)}$$

then the eqn (3) divided by yt

$$\frac{1}{y} \frac{\partial^2 y}{\partial y^2} = \frac{1}{v^2 t} \frac{\partial^2 t}{\partial T^2} \quad \text{——— (4)}$$

then constant $-k^2$

$$\frac{\partial^2 y}{\partial y^2} + k^2 y = 0$$

$$\frac{\partial^2 t}{\partial T^2} + a k^2 v^2 t = 0$$

~~then~~ the equation are,

$$X = \begin{Bmatrix} \sin kx \\ \cos kx \end{Bmatrix} \begin{Bmatrix} \sin kvt \\ \cos kvt \end{Bmatrix}$$

Or

$$X = \begin{Bmatrix} \sin ky \\ \sin ky \end{Bmatrix} \begin{Bmatrix} \sin k^2v^2t \\ \cos k^2v^2t \end{Bmatrix}$$

(2) Laplace equation.

Laplace equation: means it is a study state of rectangular plate it also called as. the Laplace "infinite Laplace equation."

~~then, the infinite~~

We can assume that in a rectangular plate of ~~ar~~ so for long separation of the width is extented the y-direction it called semi finite length"

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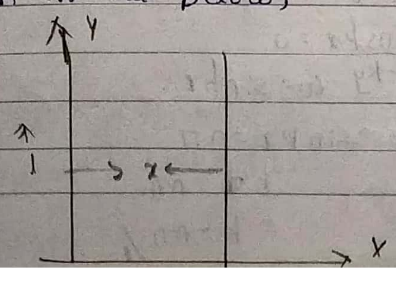
the rectangular plate,
the length is "l" and width is 'x' ~~in~~ the fig shown in the below,



Laplace equation dependu on initial temp and final temp. it we determine the it's width and length and height of the rectangular plate.

the Laplace equation, is no flow of the heat.

$$\nabla^2 T = 0 \quad \text{--- (1)}$$

then, ∇^2 = it is the rectangular plante because the side is in rectangular shape.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} = 0 \quad \text{--- (2)}$$

there, $T = X(x)\, Y(y)$

X = is function of x

Y = is function of y.

$$\frac{\partial^2 (XY)}{\partial x^2} + \frac{\partial^2 (XY)}{\partial y^2} = 0$$

$$Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} = 0$$

divided by XY

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{d^2 Y}{dy^2} = -k^2 \qquad k = \text{constant}.$$

$$\frac{\partial^2 X}{\partial x^2} + k^2 X = 0 \qquad \frac{\partial^2 Y}{\partial y^2} + k^2 Y = 0$$

$$X = \begin{Bmatrix} \sin kx \\ \cos kx \end{Bmatrix} \qquad Y = \begin{Bmatrix} e^{+ky} \\ e^{-ky} \end{Bmatrix}$$

first at temp = 0 and $y = \infty$

we discard the e^{-ky}.

and $t = 0$ $x = 0$

$\cos kx = 0$

$y = e^{-ky}$ ~~cos~~ $\sin kx$.

$x = l$, $\sin kx = n\pi$

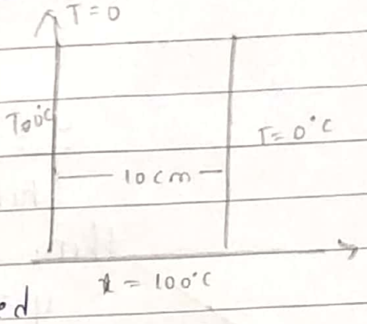
$k l = n\pi$

$k = n\pi / l$

$$\boxed{y = e^{-\frac{n\pi y}{l}} \sin kx}$$

this is Laplace equation.

(b) A rectangular metal plate has its 2 sides and.
T = 0°C , l = 100°C 2 = 10 cm
steady state temp.



then Laplace equation,
we can assume that in a,
rectangular plate so for long
separation of width is extented
the y-direction it called "semi- finite length"

the Laplace equation. there is no flow of the heat,

$$\nabla^2 T = 0 \quad \text{——— (1)}$$

∇^2 = it is a rectangular plate.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} = 0 \quad \text{——— (2)}$$

there is, function,

$$T = X(x)\ Y(y)$$

X is function of x
Y is function of y

$$\frac{\partial^2 (XY)}{\partial x^2} + \frac{\partial (XY)}{\partial y^2} = 0 \quad \text{——— (3)}$$

∴ eqⁿ (3) divided XY

$$\frac{\cancel{Y}}{X}\frac{\partial^2 x}{\partial x^2} + \frac{1}{Y}\frac{\partial (xY)}{\partial y^2} = 0$$

$$\frac{1}{X}\frac{\partial^2 x}{\partial x^2} + k^2 \qquad \frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -k^2 \qquad \frac{1}{Y}\frac{\partial (Y)}{\partial y^2} = -k^2$$

the k = constant

$$\frac{\partial^2 X}{\partial x^2} + k^2 x = 0 \qquad \frac{\partial^2 Y}{\partial y^2} + k^2 y = 0$$

$$X = \begin{Bmatrix} \sin kx \\ \cos kx \end{Bmatrix} \qquad Y = \begin{Bmatrix} e^{-ky} \\ e^{-ky} \end{Bmatrix}$$

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$t=0$ $y=\infty$, first case, e^{ky} disgard

$t=0$, $x=0$, $\cos kx$ discard

$y = e^{-ky} \sin kx$ —— (4)

and $x=10$

$\sin kx = n\pi$

$k(10) = n\pi$

$k = \frac{n\pi}{10}$

$y = e^{-\left(\frac{n\pi y}{10}\right)} \sin\left(\frac{x n\pi}{10}\right)$ —— (5)

here statisfy $t=0$.

then, $t = 100°c$. $x = 10$

$y = \sum_{n=0}^{\infty} b_n \sin\left(\frac{x n\pi}{10}\right)$

the sine fourier series,

we find the b_n,

Where $l = 10$ $f(x) = 100$

$b_n = \frac{2}{l} \int_0^l f(x) \sin\left(\frac{n\pi x}{10}\right)$

$b_n = \frac{2}{10} \int_0^l 100 . \sin\left(\frac{n\pi x}{10}\right)$

the $\begin{cases} \frac{400}{n\pi} & \text{is even} \\ 0 & \text{is odd} \end{cases}$

$y = \frac{400}{\pi} \left\{ e^{-\frac{n\pi y}{10}} \sin \frac{n\pi}{10} - e^{-3n\pi y/10} \sin \frac{3n\pi}{10} \cdots \right\}$

$\frac{\pi y}{10}$ is small, its value variable

Ex =

$x = 5$ $y = 5$

$y = \frac{400}{\pi} \left\{ e^{\pi/2} \sin(\pi/2) - e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) \cdots \right\}$

$y = 26.2°c$

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