B.L.D.E. Association's

S. B. Arts & K.C.P. Science College, VIJAYAPUR- 586 103.

ASSIGNMENT

For ~~B.A.~~/ B.Sc. ....Vth........ Semester
2022 - 2023

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Subject Mathematics -II

Assignment No.	Date	Marks Assigned	Marks Obtained	Name and Signature of Teacher	Remarks
1	26/2/23	03	03	[signature]	
2					
3					
4					

ASSIGNMENT :-

NM

1) Find the real root of $2x = \cos x + 3$ using iteration method in three stages.

→ $f(x) = 2x = \cos x + 3$

$f(x) = 2x - \cos x + 3$

$f(0) = 2(0) - \cos(0) + 3 = 2$

$f(1) = 2(1) - \cos(1) + 3 = 4.4596$

$f(2) = 2(2) - \cos(2) + 3 = 7.4161$

The root lies b/w 1 & 2

$x_0 = \frac{1+2}{2} = 1.5 \qquad x = \frac{\cos x + 3}{2}$

$\phi(x) = \frac{\cos x + 3}{2} \qquad \phi'(x) = -\sin x \Rightarrow -\sin(1.5) = -0.9974.$

by iteration is applied

$\phi'(x) = \frac{\cos x + 3}{2}.$

$x_1 = \phi(x_0) = \frac{\cos x_0 + 3}{2} = \frac{\cos(1.5) + 3}{2} = 1.53353$

$x_2 = \phi(x_1) = \frac{\cos x_1 + 3}{2} = \frac{\cos(1.53353)}{2} = 1.51771$

$x_3 = \phi(x_2) = \frac{\cos x_2 + 3}{2} = \frac{\cos(1.51771) + 3}{2} = 1.52655.$

2) Prove that $\nabla = E^{-1}\Delta$

$\nabla f(x) = f(x) - f(x-h)$

$= \Delta f(x-h)$

$= \Delta E^{-1} f(x)$

$\nabla = \Delta E^{-1}$ or $\nabla E = \Delta.$

3) Prove that $\Delta \log x [f(x)] = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$

→ $\Delta \log f(x) = \log f(x+h) - \log x$

$\log \frac{f(x+h)}{f(x)}$

$\log\left[\frac{f(x) + f(x+h) - f(x)}{f(x)}\right]$

$\log\left[\frac{f(x) + \Delta f(x)}{f(x)}\right]$

$\log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$ //

4) Evaluate $\Delta^3(1+2x)(1+4x)(1+6x)$ where $h=1$

→ $\Delta^3(1+2x)(1+4x)(1+6x)$ $h=1$

$a_n = 2 \times 4 \times 6 \Rightarrow 48$ $h=1$

$n=3$.

$\Delta^n f(x) = a_n n! h^n$

$= 48 \cdot 3! \cdot 1$

$= 48 \times 3 \times 2 \times 1$.

$\Rightarrow 288$ //.

5) Write the formula to find the first derivative using forward difference

→ The difference $f(x+h)-f(x)$ is called forward difference

$\Delta f(x) = f(x+h) - f(x)$

$\Delta f(a) = f(a+h) - f(a)$

$\Delta f(a+h) = f(a+2h) - f(a+h)$

$\Delta f(a+2h) = f(a+3h) - f(a+2h)$ etc are called first ordered forward difference

6) Explain briefly Bisection method to find the real root of the equation $f(x)=0$.

→ Let $f(a) \cdot f(b)$ are of opposite signs.

Such that $f(a)f(b)<0$.

Let x_0 be the mid-point of interval (a,b) Then app root is defined by

$x_0 = \frac{a+b}{2}$

If $f(x_0)=0$. x_0 is the desired root of $f(x)=0$.

$f(x_0) \neq 0$. The root may be in b/w $[a, x_0]$ & $[x_0, b]$

If the root lies b/w $[a, x_0]$ then $f(a) f(x_0) < 0$.

x_1 is the mid-point of (a, x_0)

$x_1 = a + x_0 / 2$.

& If the root lies b/w $[x_0, b]$

Then $f(x_0) f(b) < 0$

If x_2 is the mid point of $(x_0 b)$

$x_2 = \frac{x_0 + b}{2}$.

This process is continued until.

7) $\Delta - \nabla = \Delta \cdot \nabla$.

$\rightarrow \Delta f(x) = f(x+h) - f(x)$

$\nabla f(x) = f(x) - f(x-h)$

$LHS = \Delta \nabla f(x) = \Delta [f(x) - f(x-h)]$

$\Delta f(x) - \Delta f(x-h)$

$\Delta f(x) - [f(x) - f(x-h)]$

$\Delta f(x) - \nabla f(x)$

$\Delta \nabla = \Delta - \nabla$. //

5M

1) Express the function $f(x) = x^4 - 12x^3 + 24x^2 - 30x + 9$ & its Successive difference in a factorial notations when $h=1$

$\rightarrow$ $f(x) = x^4 - 12x^3 + 24x^2 - 30x + 9$

$x=0$	1	-12	24	-30	9
		0	0	0	0
$x=1$	1	-12	24	-30	9
		1	-11	13	
$x=2$	1	-11	13	17	
		2	-18		
$x=3$	1	-9	-5		
		3			
$x=4$	1	$\boxed{-6}$			
	1	A			

$f(x) = x^4 - 6x^3 - 5x^2 - 17x^1 + 9$.

which is Required factorial notation of $f(x)$

$f(x) = x^4 - 6x^3 - 5x^2 - 17x^1 + 9$

$\Delta y = 4x^3 - 18x^2 - 10x^1 - 17$

$\Delta^2 y = 12x^2 - 36x^1 - 10$

$\Delta^3 y = 24x^1 - 36$

$\Delta^4 y = 24$ //

(2) $f(x) = x^3 - x - 1 = 0$. root lying b/w 1 & 2.

$\rightarrow$ let $f(x) = x^3 - x - 1$.

$f(1) = 1 - 1 - 1 = -1$ (-ve)

$f(2) = 8 - 2 - 1 = 5$ (+ve)

Since $f(1) f(2) < 0$.

1st Iteration b/w 1 & 2

$x_1 = \frac{1+2}{2} = \frac{3}{2} = 1.5$

$f(1.5) = (1.5)^3 - (1.5) - 1 \Rightarrow 0.875$ (+ve)

The root lies b/w 1 & 1.5

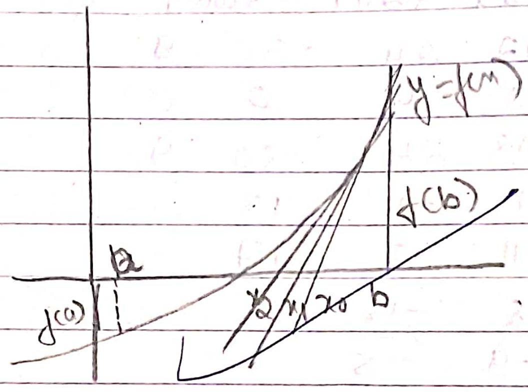
$x_2 = \frac{1+1.5}{2} = 1.25$

$f(x) = f(1.25) = (1.25)^3 - (1.25) - 1 = -0.297$ (-ve)

The root lies b/w 1.5 & 1.25

$x_3 = \frac{1.5+1.25}{2} = 1.375$ //

③. Derive the Newton - Raphson's formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$



Let x_0 be an approximate root of $f(x) = 0$.

Let $x_1 = x_0 + h$ be the corrected / Exact root of $f(x) = 0$.

where 'h' is small correction

$\therefore f(x_1) = f(x_0 + h) = 0$

Expanding $f(x_0 + h)$ by taylor's series

$f(x_0) + \frac{h}{1!} f'(x_0) + \frac{h^2}{2!} f''(x_0) + \cdots$

Neglecting the Second & Higher order derivative

we have, $f(x_0) + \frac{h}{1!} f'(x_0) = 0$

$f(x_0) = -h f'(x_0)$

$h = -\frac{f(x_0)}{f'(x_0)}$

Substitute the value of 'h' in $x_1 = x_0 + h$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Successive approximations are given by $x_2\, x_3$ —

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad //$$

(4) form of D.Eq^n by eliminating arbitrary Constant a and b from the eq^n $y_n = a2^n + b3^n$.

→ $y_n = a^{2n} + b3^n$

$y_{n+1} = a^{2n+1} + b3^{n+1}$

$y_{n+2} = a_2^{2n+2} + b3^{n+2}$.

$$\begin{vmatrix} y_n & 2^n & 3^n \\ y_{n+1} & 2^n \cdot 2 & 3^n \cdot 3 \\ y_{n+2} & 2^n \cdot 2^2 & 3^n 3^2 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} y_n & 1 & 1 \\ y_{n+1} & 2 & 3 \\ y_{n+2} & 4 & 9 \end{vmatrix} = 0.$$

$$y_n[18-12] - y_{n+1}[9-4] + y_{n+2}(3-2)$$

$$6y_n - 5y_{n+1} + y_{n+2} \quad //$$

(5) Solve D.Eq^n $(E^3 - 5E^2 + 3E + 9)y_n = 2^n + 3^n$.

→ $(E^3 - 5E^2 + 3E + 9)y_n = \cancel{2^n + 3^n}$

A.E. $E^3 - 5E^2 + 3E + 9 = 0$

-1	1	-5	3	9
		-1	6	-9
	1	-6	9	0

$E^2 - 6E + 9 = 0 \Rightarrow E^2 - 3E - 3E + 9 = 0$

$E(E-3) - 3(E-3) = 0$

$(E-3)(E-3) = 0$

$E = 3, 3.$

$C.F = C_1(-1)^x + (C_2 + C_3 x)3^x$.

$$P.I = \frac{1}{E^3 - 5E^2 + 3E + 9} 2^n + \frac{1}{E^3 - 5E^2 + 3E + 9} 3^n$$

$$\frac{2^n}{3} + \frac{1}{0} 3^n$$

$$\frac{2^n}{3} + \frac{1}{(E-3)^2 (E+1)} 3^n$$

$$\frac{2^n}{3} + \frac{1}{4} n(n-1) 3^n$$

$$P.I \quad \frac{2^n}{3} + \frac{1}{8} n(n-1)3^n$$

G.S = C.F + P.I

$G.S = C_1(-1)^x + (C_2 + (3x))3^x + \frac{2^n}{8} + \frac{1}{8} n(n-1)3^n$ //

6) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with boundary Condition $y=1$ at $x=0$

find approx value of y for $x=0.1$ by Euler's Modified Method correct to 4 decimal place with $h=0.1$.

→ Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ $y(0)=1$ $y=1$ $h=0.1$

$x_0=0$ $y_0=1$ $x_0=0$ $x_1=0.1$

Stage ① By Euler's formula.

$y_1 = y(0.1) = y_0 + hf(x_0 y_0)$

$1+0.1 f(0,1)$

$1+0.1\left(\frac{0-1}{0+0}\right) = 1+0.1\left(\frac{-1}{1}\right)$

$y^{(0)} = 0.9$

Now we can Improved it by modified E. Method Iteration

$y_1^{(1)} = y_0 + \frac{h}{2}\left(f(x_0, y_0) + f(x_1, y_1^{(0)})\right)$

$y_1^{(1)} = 1+\frac{0.1}{2}\left\{\frac{1-0}{1+0} + \frac{0.9-0.1}{0.9+0.1}\right]$

$y_1^{(1)} = 1+0.05\left[(1) + 0.8/1\right]$

$y_1^{(1)} = 1+0.09$

$y_1^{(1)} = 1.09$

2nd Iteration $y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$

$y_1^{(2)} = 1+\frac{0.1}{2}\left[\frac{1-0}{1+0} + \frac{1.09-0.1}{1.09+0.1}\right]$

$y_1^{(2)} = 1+0.05\left[1 + 0.99/1.19\right]$

$= y_1^{(2)} = 1+0.05\left[1+0.8319327\right]$

$1+0.0915966$

$y_1^{(2)} = 1.0915966$

3rd Iteration.

$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_0, y_1^{(2)})\right]$

$1+\frac{0.1}{2}\left[(1) + \frac{1.09159-0.1}{1.09159+0.1}\right]$

$1+0.05\left[(1)+0.8321570\right]$

$y_1^{(3)} = 1.09160$

4th iteration :

$$y_1^{(4)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_0, y_1^{(3)})\right]$$

$$= 1 + \frac{0.1}{2}\left[1 + \left[\frac{1.09160 - 0.1}{1.09160 + 0.1}\right]\right]$$

$$= 1 + 0.05\left[1 + \left[0.9916 / 1.1916\right]\right]$$

$$= 1 + 0.05\left[0.83215\right]$$

$$y_1^{(4)} = 1.09160$$

7) Explain picard's method to solve the equation $\frac{dy}{dx} = f(x,y)$ with initial condition $y(x_0) = y_0$.

→ Consider the 1st order Differential equation $\frac{dy}{dx} = f(x,y)$ with initial condition $y(x_0) = y_0$ — ①

eqn ① can be written as

$$dy = f(x,y)\,dx \quad \text{— ②}$$

Integrating both sides of eqn ② b/w the corresponding limit for x & y we get

$$\int_{y_0}^{y} dy = \int_{x_0}^{x} f(x,y)\,dx$$

$$[y]_{y_0}^{y} = \int_{x_0}^{x} f(x,y)\,dx$$

$$y - y_0 = \int_{x_0}^{x} f(x,y)\,dx$$

$$y = y_0 + \int_{x_0}^{x} f(x,y)\,dx \quad \text{— ③}$$

we get a first approximation for y by putting y_0 for y in the integrant of eqn ③ . which is given by.

$$y^{(1)} = y_0 + \int_{x_0}^{x} f(x,y)\,dx \quad \text{— ④}$$

Now in eqn ④

$f(x, x_0)$ is a function of x only

∴ Integral in RHS of eqn ④ can be solved easily

again if we substitute $y^{(1)}$ for y in eqn ③

we get 2nd approximation for y which is given by

$$y^{(2)} = y_0 + \int_{x_0}^{x} f(x,y)\,dx .$$

Continuous ~~this~~ process of substitute as many times as may be necessary ~~the~~ nth app. is given by

$$y^{(n)} = y_0 + \int_{x_0}^{x} f(x, y^{n-1})\, dx$$

(8). find $y(0.1)$ and $y(0.2)$ given $y' = x - 2y$, $y(0) = 1$ taking $h = 0.1$ by Second order Runge-kutta method.

→ Given $f(x, y) = x - 2y$

$x_0 = 0 \quad y_0 = 1 \quad h = 0.1$

Second order Runge-kutta method

$k_1 = hf(x_0, y_0)$

$h(x_0 - 2y_0)$

$= 0.1(0 - 2(1)) = 0.1(-2) = -0.2.$

$k_2 = hf(x_0 + h/2, y_0 + k_1/2)$

$= hf\left(0 + \frac{0.1}{2}, 1 - \frac{0.2}{2}\right)$

$0.1 f(0.05, 0.9)$

$0.1[(0.05 - 2(0.9))]$

$0.1(-1.75)$

$k_2 = -0.175$

$k_2 = \Delta y$

$y_1 = y_0 + \Delta y = 1 - 0.175 = 0.825$

for $y(0.1) = 1 - 0.175 = 0.825$

$y(0.2) = y_2 = y_1 + \Delta y_1$

$k_1 = hf(x_1, y_1)$

$0.1 f(0.1, 0.825)$

$0.1 f[0.1 - 2(0.825)]$

$k_1 = -0.155$

$k_2 = hf\left[x_1 + \frac{h}{2}, y_1 + k_1/2\right]$

$= 0.1 f\left(0.1 + \frac{0.1}{2}, 0.825 + \left(\frac{-0.155}{2}\right)\right)$

$= 0.1 f(0.15, 0.7475)$

$= 0.1[0.15 - 2(0.7475)] = -0.1345$

$k_2 = \Delta y = y_1 + \Delta y$

$= 0.825 - 0.1345$

$= 0.6905$

(9) Find the approximate root of $x\log_{10}x = 1.2$ by Newton Raphson method with $x_0 = 2.5$ correct to three decimal places.

→ Let $f(x) = x\log_{10}x - 1.2 = 0$.

$$f(x) = \frac{x\log_e x}{\log_e 10} - 1.2$$

$$f'(x) = \frac{1}{\log_e 10}\left[\log_e(x)(1) + x\frac{1}{x}\right]$$

$$= \frac{1}{\log_e 10} \quad \frac{x+1}{\log 10}$$

$$f'(x) = \log_{10}x + 0.4343$$

by Newton Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{(x_n\log_{10}x_n - 1.2)}{\log_{10}x_n + 0.4343}$$

$$x_{n+1} = \frac{\log_{10}x_n + 0.4343x_n - x_n\log_{10}x_n + 1.2}{\log_{10}x_n + 0.4343}$$

$$x_{n+1} = \frac{0.4343x_n + 1.2}{\log_{10}x_n + 0.4343}$$

1st $= \frac{0.4343x_0 + 1.2}{\log_{10}x_0 + 0.4343} \Rightarrow \frac{(0.4343(2.5) + 1.2)}{(\log_{10}(2.5) + 0.4343)}$

$$x_1 \Rightarrow 2.7465$$

2nd $= \frac{0.4343x_1 + 1.2}{\log_{10}x_1 + 0.4343} = \frac{(0.4343(2.7465) + 1.2}{\log_{10}(2.7465) + 0.4343}$

$$x_2 = 2.7406$$

3rd $x_3 = \frac{0.4343 + 1.2}{\log_{10}(x_2) + 0.4343} = \frac{(0.4343(2.7406) + 1.2}{\log_{10}(2.7406) + 0.4343}$

$$x_3 = 2.7406$$

The Required root of given equation

$$x_3 = 2.7406$$

(10) Solve the following equations by Jacobi method.

$27x+6y-z=85$

$6x+15y+2z=72$

$x+y+54z=110.$

→ The above eq^n can be written as,

$x=\frac{1}{27}(85-6y+z)$

$y=\frac{1}{15}(72-6x-2z)$

$z=\frac{1}{54}(110-x-y)$

Let us take the first approximation to be $x^{(1)}, y^{(1)}, z^{(1)}=0$
obtain second approximation from (1)

$x^{(2)}=\frac{1}{27}(85-6y^{(1)}+z^{(1)}]$

$=\frac{1}{27}(85-0+0)=\frac{85}{27}=3.15$

$y^{(2)}=\frac{1}{15}[72-6x^{(1)}-2z^{(1)}]$

$=\frac{72}{15}=4.8$

$z^{(2)}=\frac{1}{54}(110-x^{(1)}-y^{(1)}]$

$=\frac{1}{54}110 \Rightarrow 2.04.$

Now we obtain third app. as follows,

$x^{(3)}=\frac{1}{27}(85-6y^{(2)}+z^{(2)}]$

$=\frac{1}{27}[85-6(4.8)+2.04)= 2.16$

$y^3=\frac{1}{15}[72-6x^{(2)}-2z^{(2)}] =3.27$

$z^3=\frac{1}{54}[110-x^{(2)}-y^{(2)}= 1.89.$

Now fourth app. is,

$$x^{(4)} = \frac{1}{27}\left(85 - 6y^{(3)} + z^{(3)}\right) = 2.49$$

$$y^{4} = \frac{1}{15}\left(72 - 6x^{3} - 2z^{3}\right) = 3.68$$

$$z^{(4)} = \frac{1}{54}\left[110 - x^{3} - y^{3}\right] = \frac{1}{54}\left(110 - 2.16 - 3.27\right)$$

$$= 1.95$$

The fifth approx is,

$$x^{5} = \frac{1}{27}\left(85 - 6y^{(4)} + z^{(4)}\right)$$

$$= \frac{1}{27}\left(85 - 6(3.68) + 1.95\right) = 2.40$$

$$y^{5} = \frac{1}{15}\left(72 - 6x^{(4)} - 2z^{4}\right)$$

$$= \frac{1}{15}\left(72 - 6(2.49) - 2(1.95)\right) = 3.54$$

$$z^{5} = \frac{1}{54}\left(110 - x^{4} - y^{4}\right) = 1.92$$

$\therefore$ $x = 2.4$ $y = 3.54$ $z = 1.92$ ~~is~~ required solution.

(11) If $y = 2x^3 - x^2 + 3x + 1$ find 'y' corresponding to the values 0 to 5 and construct the forward difference table. Prove that 2nd forward difference is $12x + 10$.

$\rightarrow$ $y = f(x) = 2x^3 - x^2 + 3x + 1$

That is x varies from 0 to 5 with $h = 1$

x	0	1	2	3	4	5
$y = f(x)$	1	5	19	55	125	241

The forward difference table is,

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0	1					
		4				
1	5		10			
		14		12	0	
2	19		22		0	
		36		12		0
3	55		34		0	
		70		12		
4	125		46			
		116				
5	241					

Now $y = 2x^3 - x^2 + 3x + 1$

$\Delta y = \Delta(2x^3 - x^2 + 3x + 1)$

$\Delta 2x^3 - \Delta x^2 + \Delta 3x + \Delta(1)$.

$2[(x+h)^3 - x^3] - [(x+h)^2 - x^2] + 3[(x+h) - x] + 0$

$2[\cancel{x^3} + h^3 + 3xh(x+h) - \cancel{x^3}] - [\cancel{x^2} + h^2 + 2xh - \cancel{x^2}] + 3h$

$2(h^3 + 3x^2h + 3xh^2) - h^2 - 2xh + 3h$

$2h^3 + 6x^2h + 6xh^2 - h^2 - 2xh + 3h$

Here $h = 1$

$2 + 6x^2 + 6x - 1 - 2x + 3$

$\Delta y = 6x^2 + 4x + 4$.

$\Delta^2 y = \Delta(\Delta y)$

$= \Delta(6x^2 + 4x + 4)$

$\Delta 6x^2 + \Delta 4x + \Delta 4$

$6((x+h)^2 - \cancel{x^2}) + 4(\cancel{x} + h - \cancel{x}) + 0$

$\Delta^2 y = 6h^2 + 12xh + 4h$

Here $h = 1$

$= 6 + 12x + 4$

$\Delta^2 y = 12x + 10$

The Second difference is $12x + 10$

(12) If $f(x)$ be a polynomial of nth degree in x, then $\Delta^n f(x)$ is constant and $\Delta^{n+1} f(x) = 0$.

→ Statement: If $f(x)$ be a polynomial of nth degree in x, then $\Delta^n f(x)$ is constant and $\Delta^{n+1} f(x) = 0$.

Proof: Let $f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$
where $a_0, a_1, a_2 \cdots a_n$ are constants & $a_0 \neq 0$.
Now,
$f(x+h) = a_0 + a_1(x+h) + a_2(x+h)^2 + \cdots + a_n(x+h)^n]$
The forward difference table is,
$\Delta f(x) = f(x+h) - f(x)$
$[a_0 + a_1(x+h) + a_2(x+h)^2 + \cdots + a_n(x+h)^n]$
$- [a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n]$

$= a_1 h + a_2(h^2 + 2xh) + \cdots + a_n[{}^nC_1 x^{n-1} h + {}^nC_2 x^{n-2} h^2$
$\cdots + h^n]$ ${}^nC_1 = n$.

$= a_1 h + a_2(h^2 + 2xh) + a_n(n x^{n-1} h + \cdots + h^n)$
$= a_0' + a_1' x + \cdots + a_n n x^{n-1} h$

$\Delta f(x) = a_0' + a_1' x + \cdots + n a_n' h x^{n-1}$

$\Delta^2 f(x) = \Delta[\Delta f(x)]$
$a_0'' + a_1'' x + \cdots + a_n'' h^2 x^{n-2}\, n(n-1)$
which is polynomial of degree $(n-2)$
proceeding on this way we get
$\Delta^n f(x) = a_n[n(n-1)(n-2) \cdots 3\times 2\times 1]\, h^n x^0$
$\Delta^n f(x) = a_n n!\, h^n \quad \because (x^0 = 1)$
$\Delta^n f(x) =$ constant.

$\Delta^{n+1} f(x) = \Delta[\Delta^n f(x)]$
$= \Delta[\text{constant}]$
$= 0$
$\therefore \Delta^{n+1} f(x) = 0$ //

(13). Derive General quadrature formula for equidistant ordinates

→ **Statement:** let $I=\int_a^b f(x)dx$ & let $y=f(x)$ take the values $y_0, y_1, y_2 \ldots$ at $x=x_0, x_0+h, x_0+2h \ldots x_0+nh$ respectively. then.

$$I=\int_{x_0}^{x_0+nh} f(x)dx = nh\left[y_0+\frac{n}{2}\Delta y_0+\frac{1}{2}\left(\frac{n^2}{3}-\frac{n}{2}\right)\Delta^2 y_0 + \frac{1}{6}\left(\frac{n^3}{4}-n^2+n\right)\Delta^3 y_0 + \ldots\right]$$

proof: let us consider divide the interval (a,b) into n sub intervals of width 'h'.

$a=x_0, x_1, x_2 \ldots x_n=b. \in (a,b)$

$a=x_0, x_0+h, x_0+2h \ldots x_0+nh.$

then

$$I=\int_a^b f(x)dx = \int_{x_0}^{x_0+nh} f(x)dx$$

by newton's forward interpolation formula.

$$y=f(x)=y_0+\frac{u\Delta y_0}{1!}+\frac{u(u-1)}{2!}\Delta^2 y_0+\frac{u(u-1)(u-2)}{3!}\Delta^3 y_0+\ldots$$

$$u=\frac{x-x_0}{h}$$

$$du=\frac{1}{h}dx \qquad u=x_0=0$$

$$u=x_0+nh=n.$$

$$h\,du=dx.$$

from eq (1) becomes,

$$\int_{x_0}^{x_0+nh} f(x)dx=\int_0^n\left[y_0+\frac{u\Delta y_0}{1!}+\frac{u(u-1)}{2!}\Delta^2 y_0+\frac{u(u-1)(u-2)}{3!}\Delta^3 y_0+\ldots\right]h\,du$$

$$\left[uy_0+\frac{u^2}{2}\Delta y_0+\frac{1}{2}\left(\frac{u^3}{3}-\frac{u^2}{2}\right)\Delta^2 y_0+\frac{1}{3!}\left(\frac{u^4}{4}-\frac{3u^3}{3}+\frac{2u^2}{2}\right)\Delta^3 y_0\right]_0^n h$$

$$=\left[ny_0+\frac{n^2}{2}\Delta y_0+\frac{1}{2}\left(\frac{n^3}{3}-\frac{n^2}{2}\right)\Delta^2 y_0+\frac{1}{6}\left(\frac{n^4}{4}-\frac{3n^3}{3}+\frac{2n^2}{2}\right)\Delta^3 y_0\right] h$$

$$\Rightarrow \left[n y_0 + \frac{n^2}{2}\Delta y_0 + \frac{1}{2}\left(\frac{n^3}{3} - \frac{n^2}{2}\right)\Delta^2 y_0 + \frac{1}{6}\left(\frac{n^4}{4} - n^3 + n^2\right)\Delta^3 y_0\right] h.$$

$$\Rightarrow hn\left[y_0 + \frac{n}{2}\Delta y_0 + \frac{1}{2}\left(\frac{n^2}{3} - \frac{n}{2}\right)\Delta^2 y_0 + \frac{1}{6}\left(\frac{n^3}{4} - n^2 + n\right)\Delta^3 y_0 + \dots\right]$$

This equation for General Quadrature formula.

(14) Explain Gauss-Seidal Iteration method to solve the system of linear equations.
$a_1x + b_1y + c_1z = d_1$; $a_2x + b_2y + c_2z = d_2$ &
$a_3x + b_3y + c_3z = d_3$.

→ The Given System of equation can be written as,

$$\left.\begin{aligned} x &= \frac{1}{a_1}[d_1 - b_1y - c_1z] \\ y &= \frac{1}{b_2}[d_2 - a_2x - c_2z] \\ z &= \frac{1}{c_1}[d_3 - a_3x - b_3y] \end{aligned}\right\} \quad (1)$$

Now taking initial approximations as $x^{(1)} = 0\ y^{(1)}\ z^{(1)} = 0$ we get,

$$x^{(2)} = \frac{1}{a_1}[d_1 - b_1y^{(1)} - c_1z^{(1)}] = d_1/a_1 \quad (2)$$

Now using $x^{(2)}$ & $z^{(1)}$ we can find $y^{(2)}$ as

$$y^{(2)} = \frac{1}{b_2}[d_2 - a_2x^{(1)} - c_2z^{(1)}]$$

$$z^{(2)} = \frac{1}{c_3}[d_3 - a_3x^{(2)} - b_3y^{(2)}].$$

So now using $x^{(2)}, y^{(2)}, z^{(2)}$ we can find third approximation as,

$$x^{(3)} = \frac{1}{a_1}[d_1 - b_1 y^{(2)} - c_1 z^{(2)}]$$

$$y^{(3)} = \frac{1}{b_2}[d_2 - a_2 x^{(3)} - c_2 z^{(2)}]$$

$$z^{(3)} = \frac{1}{c_3}[d_3 - a_3 x^{(3)} - b_3 y^{(3)}]$$

and proceeding in this way we get nth approximations,

$$x^{(n)} = \frac{1}{a_1}[d_1 - b_1 y^{(n-1)} - c_1 z^{(n-1)}]$$

$$y^{(n)} = \frac{1}{b_2}[d_2 - a_2 x^{(n)} - c_2 z^{(n-1)}]$$

& $$z^{(n)} = \frac{1}{c_3}[d_3 - a_3 x^{(n)} - b_3 y^{(n)}]$$

proceed this way until we get solution to required accuracy.

B.L.D.E. Association's

S. B. Arts & K.C.P. Science College, VIJAYAPUR- 586 103.

ASSIGNMENT

For B.A./ B.Sc. ..VIth.......... Semester

2021 - 2022

Name of the Student Sampath Hugar

Roll No. 01 R.C.U. Seat No. S1925187.

Subject Mathematics - I.

Assignment No.	Date	Marks Assigned	Marks Obtained	Name and Signature of Teacher	Remarks
1	21/08/22	03	03		
2					
3					
4					

1) Solve $\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{(x^2+y^2)}$

$$\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{(x^2+y^2)} \rightarrow (1)$$

Choose multiplies as $-x, y, z$ each fraction,

$$\frac{-x \cdot dx + y \cdot dy + z \cdot dz}{-x^2 dz - xzy + xyz - y^2 z + x^2 z + y^2 z}$$

$$-x \cdot dx + y \cdot dy + z \cdot dz = 0.$$

Integration.

$$-\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = \frac{c_1}{2}$$

$$\boxed{-x^2 + y^2 + z^2 = c_1}$$

Consider.

$$\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)}$$

$$\frac{dx}{x+y} = \frac{dy}{x-y}$$

$$(x-y) \cdot dx = dy\,(x+y).$$

$$x \cdot dx - y \cdot dy = (y \cdot dx + x \cdot dy) = 0.$$

$$x \cdot dx - y \cdot dy - d(xy) = 0.$$

Integrating

$$\frac{x^2}{2} - \frac{y^2}{2} - xy = c_2$$

$$\boxed{x^2 - y^2 - 2xy = c_2}$$

2) Solve $\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$

$$\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy} \rightarrow (1).$$

Rewrite Eq^n as

$$\frac{dx-dy}{x^2-yz-y^2+zx} = \frac{dy-dx}{y^2-zx-x^2+xy} = \frac{dz-dx}{z^2-xy-x^2+yz}$$

$$\frac{dx-dy}{x^2-y^2+z(x-y)} = \frac{dy-dx}{y^2-z^2+x(y-z)} = \frac{dz-dx}{z^2-x^2+y(z-x)}$$

$$\frac{dx-dy}{(x-y)(x+y)+z(x-y)} = \frac{dy-dz}{(y+z)(y+z)+x(y-z)} = \frac{dz-dx}{(z-x)(z+x)+y(z-x)}$$

Consider $\dfrac{dx-dy}{(x-y)(x+y+z)} = \dfrac{dy-dz}{(y-z)(x+y+z)}$

Integrating

$(y-z)(dx-dy) = (x+y)(dy-dz)$

$\log(x-y) = \log(y-z) + \log c.$

$\log(x-y) - \log(y-z) = \log c$

$\log\left(\dfrac{x-y}{y-z}\right) = \log c$

$\dfrac{x-y}{y-z} = c_1$

Consider $\dfrac{dy-dz}{(y-z)(x+y+z)} = \dfrac{dz-dx}{(z-x)(x+y+z)}.$

$\dfrac{dy-dz}{y-z} = \dfrac{dz-dx}{z-x}.$

$\log(y-z) = \log(z-x) + \log c_2$

$\log(y-z) - \log(z-x) = \log c_2$

$\log\left(\dfrac{y-z}{z-x}\right) = \log c_2$

$\left(\dfrac{y-z}{z-x}\right) = c_2.$

(3) Solve $(y+z)\cdot dz + dy + dz = 0$

$(y+z)\cdot dx + dy + dz = 0 \rightarrow (1).$

divide eqn $-(1)$ by $(y+z).$

$dx + \dfrac{1}{y+z}\cdot dy + \dfrac{1}{y+z}\cdot dz = 0 \rightarrow (2)$

here $p=1$, $Q=\frac{1}{y+2}$ $R=\frac{1}{y+2}$.

From the integrating Condition.

$$P\left(\frac{\partial Q}{\partial z}-\frac{\partial R}{\partial y}\right)+Q\left(\frac{\partial R}{\partial x}-\frac{\partial P}{\partial z}\right)+R\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right)=0.$$

$$1\left(\log(y+x)-\log(y+2)\right)+\frac{1}{y+2}(0-0)+\frac{1}{y+2}(0-0)=0.$$

$$0=0.$$

The given Eq^n Satisfies integrability Condition.
from - (2).

$$dx+\frac{1}{y+2}\cdot dy+\frac{1}{y+2}\cdot dz=0.$$

Integrablity we get.

$$dx+\frac{dy+dz}{y+2}=0.$$

$$x+\log(y+2)=\log c_{\#}.$$

4) Solve $yz\cdot dx+xz\cdot dy+xy\cdot dz=0$.

$$yz\cdot dx+xz\cdot dy+xy\cdot dz=0 \rightarrow (1).$$

$p=yz$, $Q=xz$ $R=xy$.

From the Integrablity Condition.

$$P\left[\frac{\partial Q}{\partial z}-\frac{\partial R}{\partial y}\right]+Q\left(\frac{\partial R}{\partial x}-\frac{\partial P}{\partial z}\right)+R\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right)=0.$$

$$\cancel{1\left[\log(y+2)-\log(y+2)\right]+\frac{1}{y+2}(0-0)+\frac{1}{y+2}\cdot(0-0)=0.}$$

$$\cancel{0=0}.$$

$$yz(x-x)+xy(y-y)+xy(z-z)=0.$$

$$0=0.$$

The Eq^n. Satisfies integrability $Cond^n$ from Eq-(1).

$$yz\cdot dx+xz\cdot dy+xy\cdot dz=0.$$

$$d(xyz)=0.$$

Integrablity we get.

$$xyz=c_{\#}.$$

5] $2yz\cdot dx + zx\cdot dy - xy\,(1+z)\cdot dz = 0.$

The Eq^n is of the form
$P\cdot dx + Q\cdot dy + R\cdot dz = 0.$

$P = 2yz \qquad Q = zx \qquad R = -xy\,(1+z)$
$R = -xy - xyz$

From integrability condition,
$$P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0.$$

$2yz\,(x + x + xz) + zx\,(-y - yz - 2y) + (-xy - xyz)(2z - z)$
$2yz\,(2x + xz) + zx\,(-yz - 3y) + (xyz + xyz^2 - 2xyz^2$
$+ xyz^2$
$4xyz + 2xyz^2 - xyz^2 - 3xyz - 2xyz + xyz - 2xyz^2$
$+ xyz^2 = 0$
$0 = 0.$

The Eq^n satisfies integrability condition from ①.
$2yz\cdot dx + zx\cdot dy + xy\cdot dz - xyz\cdot dz = 0.$
divide by xyz.
$$\frac{2}{x}\cdot dx + \frac{1}{y}\cdot dy - \frac{dz}{z} - dz = 0.$$
Integrating we get.
$2\log x + \log y - \log z - z = c.$
$\log x + \log y - \log z - \cancel{z} = c + z.$
$\log x^2 y - \log z = z + c$
$$\log\left(\frac{x^2 y}{z}\right) = z + c.$$
$$\frac{x^2 y}{z} = (z+c)\cdot e^{c+z}$$
//.

6) Solve $(2x^2 + 2xy + 2xz^2 + 1)\cdot dx + dy + 2z\cdot dz = 0.$
Given Eq^n is
$(2x^2 + 2xy + 2xz^2 + 1)\cdot dx + dy + 2z\cdot dz = 0 \rightarrow ①$

Which is of the form.

$P \cdot dx + Q \cdot dy + R \cdot dz = 0.$

here.

$P = 2x^2 + 2xy + 2xz^2 + 1, \quad Q = 1, \quad R = 2z$

Condition for integrability is.

$$P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0.$$

$$(2x^2 - 2xy + 2xz^2 + 1)(0 - 0) + 1(0 - 4xz) + 2z(2x - 0) = 0$$

$$0 = 0$$

Hence the Eq^n (1) is integrable.

from -(1),

$$(2x^2 + 2xy + 2xz^2 + 1) \cdot dx + dy + 2z \cdot dz = 0,$$

Taking then Eq^n (1) x, as Constant.

then Eq-(1) becomes.

$$dy + 2z \cdot dz = 0,$$

$$\int dy + 2\int z \cdot dz = \phi(x)$$

$$y + \frac{2z^2}{2} = \phi(x).$$

$$(y + z^2) = \phi(x).$$

differentiate totally we get.

$$dy + 2z \cdot dz = \phi'(x) \cdot dx.$$

$$-\phi x \cdot dx + dy + 2z \cdot dz = 0.$$

Comparing the Eq^n (1) & (3) we get.

$$-\phi'(x) = 2x^2 + 2xy + 2xz^2 + 1$$

$$-\frac{d\phi}{dx} = 2x^2 + 2x(y + z^2) + 1$$

$$-\left(\frac{d\phi}{dx}\right) = 2x^2 + 2x\phi + 1.$$

$$\frac{d\phi}{dx} + (2x)\phi = -2x^2 - 1$$

which is linear Eq^n of the form.

$$\frac{dy}{dx} + Py = Q.$$

here $p = 2x$, $Q = -2x^2 - 1$, $y = \phi$

$I.F = e^{\int p \cdot dx}$

$= e^{\int 2 \cdot dx}$

$= e^{x^2}$

$\therefore$ The solution is

$y(I.F) = \int Q \cdot (I.F) \cdot dx + c.$

$\phi (e^{x^2}) = \int (-2x^2 - 1) e^{x^2} \cdot dx + c$

$= -\int 2x^2 \cdot e^{x^2} \cdot dx - \int e^{x^2} dx + c$

$= -\int x (2x \cdot e^{x^2}) dx - \int e^{x^2} dx + c$

$= -x \cdot e^{x^2} + \int e^{x^2} dx - \int e^{x^2} dx + c$

$= -x \cdot e^{x^2} + c.$

$\phi (e^{x^2}) + x \cdot e^{x^2} - c = 0,$

$e^{x^2} (\phi + x) - c = 0.$

$\phi = -x + c \cdot e^{-x^2}$

Put this in Eq-(2), we get-

$y + z^2 = -x + e^{-x^2} c$

$x + y + z^2 = e^{-x^2} c.$

Which is required solution of the Equation

7) Solve $z^2 dx + (z^2 - 2yz) \cdot dy + (2y^2 - yz - xz)\, dz = 0.$

Given

$z^2 dx + (z^2 - 2yz) \cdot dy + (2y^2 - yz - xz) \cdot dz = 0 \rightarrow (1)$

From Auxillary Eq^n

$$\frac{dx}{\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right)} = \frac{dy}{\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right)} = \frac{dz}{\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)}.$$

$$\frac{dx}{2z - 2y - 4y + z} = \frac{dy}{-z - 2z} = \frac{dz}{(0 - 0)}$$

$$\frac{dx}{3x - 6y} = \frac{dy}{-3z} = \frac{dz}{0}.$$

Consider last 2 ratio.

$$\frac{dy}{-3z} = \frac{dz}{0}$$

$dz = 0$

$\boxed{z = u}$

Consider

$$\frac{dx}{3z-6y} = \frac{dy}{-3z}$$

$-3z(dx) = dy(3z-6y)$

Integrating

$-3zx = 3zy - 3y^2 + c$

$\cancel{\therefore -3zx - 3zy + 3y^2 = c}$

$xz + yz - y^2 = c/3$

$\boxed{xz + yz - y^2 = v/3}$

$z = u$ & $xz + yz - y^2 = v/3$

$dz = du$ & $x \cdot dz + z \cdot dx + y \cdot dz + z \cdot dy - 2y \cdot dy = dv/3$

putting these values in below Eq^n

$A \cdot du + B \cdot dv = 0$

$A \cdot dz + 3B(x \cdot dz + z \cdot dx + y \cdot dz + z \cdot dy - 2y \cdot dy] = 0$

$3Bz \cdot dx + (3Bz - 6By) dy + (A + 3Bx + 3By) \cdot dz = 0 \rightarrow (3)$

Eq - (1) & Eq - (2) we get

$3Bz = z^2$ and $3Bz - 6By = z^2 - 2yz$

$A + 3Bx + 3By = 2y^2 - yz - xz$

$B = \frac{z}{3}$ & $A = 2y^2 - yz - xz - 3B(x+y)$

$A = 2y^2 - yz - xz - 3(z/3)(x+y)$

$A = 2y^2 - yz - xz - z(x+y)$

$B = z/3$ & $A = 2y^2 - 2yz - 2xz$

$B = u/3$ & $A = -2v/3$

put these values in $A \cdot du + B \cdot dv = 0$

$-\frac{2v}{3} \cdot du + \frac{u}{3} \cdot dv = 0$

$\div$ by uv

$$-\frac{2}{3u}\cdot du + \frac{1}{3v}\cdot dv = 0.$$

Integrating.

$$-\frac{2}{3}\log u + \frac{1}{3}\log v = \log c.$$

$$-2\log u + 1\log v = 3\log c.$$

$$u^2 v = c.$$

$$\frac{3(x^2+y^2-y^2)}{z^2} = c.$$

8) Solve $9x(1-x)y'' - 12y' + 4y = 0$ by Frobenius method.

here.

$$p(x) = \frac{-12}{9x(1-x)} \qquad q(x) = \frac{4}{9x(1-x)}.$$

Both $p(x)$ & $q(x)$ are not analytic at $x=0$.

$\therefore$ $x=0$ is Singular point.

$$x\,p(x) = \frac{-12x}{9x(1-x)} = \frac{-12}{9(1-x)}.$$

$$x^2 q(x) = \frac{4x^2}{9x(1-x)} = \frac{4x}{9(1-x)}.$$

Both $xp(x)$ & $x^2 q(x)$ are analytic at $x=0$

$\therefore$ The pt $x=0$ is a r.s.p of Eq-1

$$y = \sum_{m=0}^{\infty} C_m\, x^{m+k}, \quad C_0 \neq 0 \rightarrow (2).$$

$$y' = \sum_{m=0}^{\infty} C_m (k+m)\cdot x^{m+k-1}$$

$$y'' = \sum_{m=0}^{\infty} C_m (k+m)(k+m-1)\, x^{m+k-2} \rightarrow (3)$$

Now Substitute all these values in Eq-1 we get

$$9x(1-x)\sum_{m=0}^{\infty} C_m (k+m)(k+m-1)x^{m+k-2} - 12\sum_{m=0}^{\infty} C_m (k+m)$$
$$x^{m+k-1} + 4\sum_{m=0}^{\infty} C_m x^{m+k} = 0.$$

$$\sum C_m \left(9(k+m)(k+m-1) - 12(k+m)\right] x^{k+m-1} + \sum_{m=0}^{\infty} C_m$$
$$\{4 - 9(k+m)(k+m-1)\} x^{k+m} = 0.$$

$$9(k+m)(k+m-1) - 12(k+m) = 3(k+m)(3k+3m-7)$$
$$4 - 9(k+m)(k+m-1) = 4 - 9(k+m)^2 + 9(k+m)$$
$$= -[9(k+m)^2 - 9(k+m) - 4]$$
$$= -[9(k+m)^2 - 12(k+m) + 3(k+m-4)]$$
$$= -(3k-m-4)(3k+3m+1).$$

$$3\sum_{m=0}^{\infty} C_m (k+m)(3k+3m-7)x^{m+k-1} - \sum_{m=0}^{\infty} C_m (3k+m-4)$$
$$(3k+3m+1)x^{k+m} = 0$$

Equating to zero Co-efficient of lowest power of x namely x^{k-1}

$$3C_0 k(3k-7) = 0$$

which is Indicial Eqn.

$$k(3k-7) = 0, \quad \therefore C_0 \neq 0.$$
$$\Rightarrow k = 0 \text{ (or) } 3k-7 = 0.$$
$$\Rightarrow k = 7/3$$

The roots are distinct and don't differ by an integer.

To Obtain reaccurence reln Equate to zero the Co-efficient of x^{m+k-1}.

$$3C_m (k+m)(3k+3m-7) - C_{m-1}(3k+3(m-1)-4)[3k+3(m-1)+1] = 0.$$

$$C_m = \frac{3k+3m-2}{3(k+m)} C_{m-1} \rightarrow \text{reaurrence reln} \rightarrow (1).$$

Taking $m = 1, 2 \cdots$ in (4).

$$c_1 = \frac{3k+1}{k+1} \cdot \frac{c_0}{3}.$$

$$c_2 = \frac{3k+4}{3(k+2)} \cdot c_1$$

$$= c_0/3^2 \cdot \frac{(3k+1)}{(k+1)} \frac{(3k+4)}{(k+2)}$$

putting these values in Eq (2), we get.

$$y = x^k (c_0 + c_1 x + c_2 x^2 + \cdots).$$

$$y = c_0 x^k \left[1 + \frac{1}{3} \frac{(3k+1)}{(k+1)} x + \frac{1}{3^2} \frac{(3k+1)}{(k+1)} \frac{(3k+4)}{(k+2)} x^2 + \cdots\right.$$

put $k = 0$.

$$y_1 = c_0 \left[1 + \frac{1}{3} x + \frac{1}{3^2} \cdot \frac{4x^2}{2} + \cdots\right]$$

$$= c_0 \left[1 + \frac{1}{3} x + \frac{1}{3^2} \cdot 2x^2 + \cdots\right]$$

$$= c_0 y_0.$$

$$= c_0 u.$$

$k = 7/3$

$$y = c_0 \cdot x^{7/3} \left[1 + \frac{1}{3} \cdot \frac{(3 \cdot 7/3 + 1)}{(7/3 + 1)} \cdot x + \frac{1}{3^2} \left(\frac{(3 \cdot 7/3 + 1)(3 \cdot 7/3 + 4)}{(7/3 + 1)(7/3 + 2)} \cdot x^2\right)\right.$$

① $z - px - qy = p^2 + q^2$

Given $f(x,y,z,p,q) = z - px - qy - p^2 - q^2 = 0$.

Charpits $Eq^n = \dfrac{dx}{-\frac{\partial f}{\partial p}} = \dfrac{dy}{-\frac{\partial f}{\partial q}} = \dfrac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \dfrac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}}$

$= dq / \partial f/\partial y + q \cdot \partial f/\partial z$

$\dfrac{\partial f}{\partial p} = -x - 2p$ $\quad \dfrac{\partial f}{\partial q} = -y - 2q$ $\quad \dfrac{\partial f}{\partial x} = -p$ $\quad \dfrac{\partial f}{\partial z} = 1$

$\partial f/\partial y = -q$

$$\frac{dx}{x+2p} = \frac{dy}{y+2q} = \frac{dz}{-p(-x-2p)-q(-y-2q)} = \frac{dp}{-p+p(1)}$$

$$= \frac{dq}{-q+q(1)}.$$

$$\frac{dx}{x+2p} = \frac{dy}{y+2q} = \frac{dz}{(xp+p^2)+qy+2q^2} = \frac{dp}{0} = \frac{dq}{0}.$$

Taking 4th & 5th fractions.

$$\frac{dp}{0} = \frac{dq}{0}.$$

$\int dp = 0$ $\qquad \int dq = 0.$

Integration $\qquad$ Integration

$p = a$ $\qquad q = b.$

$dz = p \cdot dx + q \cdot dy$

$dz = a \cdot dx + b \cdot dy.$

$z = ax + by + c.$

2] $px+qy=pq$

which is of the form.

$f(x,y,z,p,q)=0$

$px+qy-pq=0$

$px+qy=pq$.

Charpits Eq^n

$$\frac{dx}{-\frac{\partial f}{\partial p}}=\frac{dy}{-\frac{\partial f}{\partial q}}=\frac{dz}{p.\frac{\partial f}{\partial p}-q.\frac{\partial f}{\partial q}}=\frac{dp}{\frac{\partial f}{\partial x}+p.\frac{\partial f}{\partial z}}=\frac{dq}{\frac{\partial f}{\partial y}+q.\frac{\partial f}{\partial z}}$$

$$\frac{dx}{-(x-q)}=\frac{dy}{\cancel{-(y-p)}}=\frac{dz}{-p(x-q)-q(y-p)}=\frac{dp}{p+p(0)}$$

$$=\frac{dq}{q+q(0)}$$

$$\frac{dx}{q-x}=\frac{dy}{p-y}=\frac{dz}{-(px-qy)+2pq}=\frac{dp}{p}=\frac{dq}{q}.$$

choosing 4th & 5th fraction

$dp/p=dq/q$.

on integration

$$\int\frac{dp}{p}=\int\frac{dq}{q}.$$

$\log p=\log q+\log a$.

$\boxed{p=qa}$

from (1) $qax+qy-aq^2=0$.

$q(ax+y)=aq^2$.

$$\boxed{q=\frac{ax+y}{a}}$$

$dz=p.dx+q.dy$.

$dz=aq.dx+\left(\frac{ax+y}{a}\right).dy$

$$z = aq \cdot dx + \frac{ax \cdot y}{a} + \frac{y^2}{2a},$$

$$= a\left(\frac{qx+y}{a}\right)dx + \frac{y^2}{2a} + xy$$

$$= \frac{ax^2}{2} + yx + xy + \frac{y^2}{2a},$$

(or)

$$dz = a\left(\frac{ax+y}{a}\right) \cdot dx + \left(\frac{ax+y}{a}\right)dy$$

$$dz = (ax+y)\,dx + \left(\frac{ax+y}{a}\right)dy.$$

on Integration

$$\int dz = a\int x \cdot dx + \int y \cdot dx + \int x \cdot dy + \frac{1}{a}\int y \cdot dy.$$

(or)

$$\int dz = \int (ax+y) \cdot \left(dx + \frac{dy}{a}\right).$$

$$\int dz = \int\left(\frac{ax+y}{a}\right)\left(a \cdot dx + \frac{dy}{a}\right).$$

$$\int dz = \int \frac{ax+y}{2a} \frac{t}{a} \cdot dt.$$

$$z = t^2/2a + b.$$

$$z = \frac{(ax+y)^2}{2a} + b \quad \#.$$

3) $q = px + q^2$

which is of the form $f(x, y, z, p, q) = 0$.

$px + q^2 - q = 0.$

Charpits Eq^n

$$\frac{dx}{-x} = \frac{dy}{2q-1} = \frac{dz}{-px-q(2q-1)} = \frac{dp}{p+p(0)} = \frac{dq}{0+q(0)}$$

$$\frac{dx}{-x} = \frac{dy}{2q-1} = \frac{dz}{-px-2q^2+q} = \frac{dp}{p} = \frac{dq}{0}$$

Choosing 4^{th} & 5^{th} fraction

$$\frac{dp}{p} = \frac{dq}{0}$$

$$\int \frac{dp}{p} = 0$$

$$\int dq = 0$$

$$\boxed{q=a}$$

$$q = px + q^2$$

$$a = px + a^2$$

$$a - a^2 = px$$

$$\frac{a-a^2}{x} = p$$

$$\frac{a(-a+1)}{x} = p$$

$$dz = p\cdot dx + q\cdot dy$$

$$dz = \frac{a(1-a)\cdot dx}{x} + a\cdot dy$$

on integration,

$$z = a(1-a)\log x + ay + b$$

4) $p = (2y+z)^2$

which is of the form $f(x,y,z,p,q)=0$

$$p = q^2y^2 + z^2 + 2qyz$$

$$q^2y^2 + z^2 + 2qyz - p = 0$$

Charpit Eq^n

$$\frac{dx}{t} = \frac{dy}{-[2qy^2+2yz]} = \frac{dz}{-p[(-1) - q(2qy^2+2yz)]}$$

$$= \frac{dp}{0+p(2z+2qy)} = \frac{dq}{2p^2y+2qz+p(2z+2qy)}$$

$$\frac{dx}{1} = \frac{dy}{-2qy^2-2yz} = \frac{dz}{p+2q^2y^2-2qyz} = \frac{dp}{2pz+2qpy}$$

$$= \frac{dq}{2q^2y+2qz+2pz+2pqy}$$

taking ② th & ④ th eqn. fraction.

$$\frac{dy}{(-2qy-2z)y} = \frac{dp}{2p(qy+z)}.$$

$$\frac{dy}{-\cancel{2}(qy+z)y} = \frac{dp}{2p(qy+z)}.$$

$$\frac{dy}{-y} = \frac{dp}{p}.$$

$$\log py = a.$$

$$\boxed{p = \frac{a}{y}}$$

$$\frac{a}{y} = (2y+z)^2$$

$$\frac{\sqrt{a}}{\sqrt{y}} = 2y+z.$$

$$\frac{\sqrt{a}}{\sqrt{y}} - z = qy$$

$$q = \frac{\frac{\sqrt{a}}{\sqrt{y}} - z}{y}.$$

$$dz = p\,dx + q\,dy$$

$$dz = \frac{a}{y} \cdot dx + \frac{\left(\sqrt{\frac{a}{y}} - z\right) \cdot dy}{y}$$

$$y \cdot dz = a \cdot dx + \left(\sqrt{\frac{a}{y}} - z\right) \cdot dy$$

$$zy = ax + 2\sqrt{a} \cdot \sqrt{y} - ay + c$$

(or)

$$y \cdot dz + z \cdot dy = a \cdot x + \frac{\sqrt{a}}{\sqrt{y}} \cdot dy$$

$$d(zy) = ax + 2\sqrt{a} \cdot \sqrt{y} + c$$

$$yz = ax + 2\sqrt{ay} + c$$ #